N.A. Rynin

INTERPLANETARY FLIGHT AND COMMUNICATION

Volume III, No. 7

K. E. Tsiolkovskii
Life, Writings, and Rockets

TRANSLATED FROM RUSSIAN

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INTERPLANETARY FLIGHT
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(Mezhplanetnye soobshcheniya)

Volume III, No. 7

K. E. TSIOLKOVSKI

Life, Writings, and Rockets

(K. E. Tsiolkovskii, ego zhizn', raboty i rakety)

Leningrad 1931

Translated from Russian

Israel Program for Scientific Translations
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PREFACE

This volume is the seventh in a series of books published separately by the author under the general title "Interplanetary Flight and Communication."

Six books have been published so far:
5. Superaviation and Superartillery. Leningrad. 1929.

The next two issues:
8. Theory of Space Flight, and
9. Astronavigation. Annals and Bibliography

are now ready for the printers, but because of financial reasons the exact date of publication is uncertain.

Finally, issue No. 10, the translation of Oberth's book "Wege zur Raumschiffahrt" (3rd edition) has been accepted by the Government Publishing House for publication.

Issue 7 dealing with the life and work of K. E. Tsiolkovskii has been included in the present series because of Tsiolkovskii's great contribution to interplanetary travel. He was the first to develop the theory of rocket flight and a substantial part of the book deals with Tsiolkovskii's work in this field.

Comments on the volumes already published and orders for additional copies should be directed to the author:

Nikolai Alekseevich Rynin, 37, Kolomenskaya ul., ap. 25, Leningrad.

Leningrad,
1 April 1931
INTRODUCTION

Konstantin Eduardovich Tsiolkovskii, a Russian scientist and inventor, is known to many as a highly original researcher in different fields of science and technology. Despite his advanced age (he was born in 1857), he is still active in his research in Kaluga. His numerous publications are widely circulated in the USSR. Some of them have been translated into European languages. There is still much left to be done, as he himself maintains. Although it is very difficult for contemporaries to assess correctly and objectively the work and the life of a genius working in their midst, we believe that a description of Tsiolkovskii's life and a brief review of his major work will be of considerable interest to wide circles of readers, despite the inevitably fragmentary and somewhat subjective treatment.

This book is published on the occasion of the 75th anniversary of K. E. Tsiolkovskii, due to take place in 1932.

N. Rynin
Chapter I

THE BIOGRAPHY OF K. E. TSIOLKOVSKII

GENERAL REMARKS

Konstantin Eduardovich Tsiolkovskii belongs to that rare category of people who devote their life to a cherished idea, and in spite of difficult material conditions and moral setbacks, remain true to it and continue working in their field of interest, even suffering great hardships. The scope of Tsiolkovskii's activities is very wide — he worked on problems of aeronautics, aerodynamics, physics, astronomy, interplanetary travel, natural sciences; he was also a writer and a philosopher.

One is impressed by his boundless energy; even in his old age he kept up his endeavors in different fields of science. Tsiolkovskii's life and work vividly demonstrate how creative a person can be, in spite of highly adverse circumstances.

The bulk of Tsiolkovskii's work, especially in recent times, seems to have been devoted to the construction of a metal airship and of an interplanetary rocket ship. Nevertheless, we think that his work on the theory of flight and his experiments in aerodynamics are no less, and perhaps even more, important. He is supposed to be the first in Russia to have built a wind tunnel and to have conducted in it a series of interesting experiments on the determination of the air resistance of various bodies (in 1891).

Tsiolkovskii's work "The Aeroplane or the Aviform (Aeronautical) Flying Machine," published in 1895, 8 years before the first trials of the Wright brothers, shows how he had foreseen the significance and basic principles of aeronautics.

His work on the design of airship hulls and testing of the design in the laboratory by hydrostatic methods (1893) is considered classical to this day.

In all his works, Tsiolkovskii was original and true to himself. And even though his circumstances in Kaluga, the city in which he spent most of his life, did not afford him the possibility of following the literature in his fields of interest, he managed to anticipate many European scientists in certain problems, and in others he arrived independently at results similar to theirs.

We do not aim to give Tsiolkovskii's complete biography here — he has much to accomplish yet; we only present his autobiography, which he provided at our request for this book. The autobiography is preceded by the letter he sent us with it.
K. TSIOLOKOVSKII'S LETTER TO N. A. RYNIN
(11 June, 1926)

Highly Esteemed Nikolai Alekseevich:

I am sending you my autobiographical notes. They might not meet with your approval, but there is nothing else I can offer you, don't even ask me. I certainly appreciate your kind intentions and am grateful to you. Enclosed is a picture of myself. It was taken in the summer of 1924. You might also use this letter as an autograph for your article about my work.

A brass model of my airship hull has been built in Moscow to my design. It occupies the big workshop in the Communist University. It is quite impressive and makes me believe in the feasibility of a metal-made dirigible. The model is 10 meters long and 2 meters high. I really do not know if this work will continue.

With utmost respect, I remain

K. Tsiolkovskii

THE AUTOBIOGRAPHY OF K. E. TSIOLOKOVSKII **

I was born on September 5th, 1857, in the village Izhevsk in the Spassk County of the Ryazan Province. My parents were poor. My father was a failure – an inventor and a philosopher. My mother, as father used to say, possessed the spark of talent. On my mother's side of the family there were some very able people. I was about 8–9 years old when my mother showed us children a balloon made from collodion. It was tiny, inflated with hydrogen, and amused me at the time as a toy.

When I was nine years old, I lost my hearing after having been sick with scarlet fever, and was regaining it very slowly. I would often get into awkward situations with children of my age and in company, and, of course, my deafness made me appear funny. This alienated me from people, and out of sheer boredom I began to read, to withdraw into myself and to daydream. I had been hurt in my self-esteem and sought to assert myself. I fondly entertained thoughts of great deeds, and at the age of eleven, I started writing silly poems.

* Concerning the publication of his works on interplanetary travel and his biography, as given in this book.
** Compiled on the basis of the following materials: a) his personal letter to us from June 11th, 1926; b) the autobiographical notes in his book, "A Simple Theory of Airships," Kaluga, 1924; c) the foreword to his book "Beyond Earth," Kaluga, 1920; and d) the biography in the book "Cosmic Rocket Ships," Kaluga, 1926, p.3.
† The inscription on the photograph he sent indicates he was born in 1857 (the inscription reads: summer 1924, at the age of 57). See also "Otviki Literaturnye" (Literary Gazette), p.25.
‡ His father was a forester.
At 14, I acquired some theoretical knowledge about air balloons from the physics of Gano. I tried to inflate a bag of tissue paper with hydrogen, but this experiment failed. I think it was at that time that I got very interested in mechanical flight with the aid of wings. I also made rudimentary lathes, which could be used for turning things; I made all sorts of machines, among which was a vehicle designed to be driven around by the wind. The model worked beautifully, and I put it on the roof, where it moved on a board against the wind. My father was very pleased, and they used to drag me off the roof, so I could show the model in the room to our guests. The experiment was a success there, too. I made the wind with the aid of bellows.

After that I started building a vehicle for my own travels. I used my breakfast money to buy nails and assorted odds and ends. But this venture was a dismal failure—partly because I ran short of patience and materials; partly because I got tired of going hungry; and partly because I realized that the whole project was not going to work anyway.

At the same time, I had another model moving on the floor—a vehicle powered by a steam-turbine engine. I did not then devote much time to aerostatics, but at the age of 15–16 I became acquainted with elementary mathematics, and after that could go into physics more thoroughly. I was particularly interested in balloons, and was able to calculate what size an air balloon with a metal hull of given thickness had to be in order to carry people aloft. It was clear to me that the thickness of the hull could be increased indefinitely when the balloon was made larger and larger. From then on the idea of a metal dirigible did not leave me. At times I tired of it, and then for months I would indulge in other subjects; but, I always used to return to it again. I did not study systematically, especially later on—I read only whatever could help me solve the problems that interested me and which I thought important. I thus got interested in the theory of the centrifugal force, because I believed it could be used to lift a spacecraft from the earth. There was a time when I thought I had found a solution to the problem (I was 16 years old then). I was so worked up that I couldn't sleep all night—I wandered about the streets of Moscow, pondering the profound implications of my discovery. But by morning I saw my invention had a basic flaw. My disappointment was as strong as my exhilaration had been. That night had a lasting impact on me. Now, thirty years later, I still have dreams in which I fly up to the stars in my machine, and I feel as excited as on that memorable night.

I also devoted some efforts to a perpetual motion machine, but I fortunately discovered within a few hours that I was proceeding from an improper understanding of magnetism.

The idea of traveling into outer space constantly pursued me. It prompted me to study higher mathematics. In 1895 I cautiously expressed my ideas on the subject in the paper "Reflections on Earth and Heaven" and later (in 1898) in the work "The Exploration of Space with Rocket-Propelled devices," which was published in "Nauchnoe Obozrenie" (Scientific Review) (No. 5, 1903).

Astronomy fascinated me because I believed, as I do even now, that someday man would reach out for the stars. My story "On the Moon" and my articles "Gravitation as a Source of Universal Energy" and others prove my unflagging interest in astronomy.
Not many books were available at the time, and few of those were accessible to me. As a result I had to work out things on my own, though I often turned out to be wrong. More than once I found I had discovered things already long known. Thus, in 1881, I developed the theory of gases, unaware of the fact that I was 24 years behind the times. But I developed my reasoning and critical faculties. In fact, I think I have essentially always tended to be an original. My loss of hearing and the social isolation it imposed on me only contributed to my independent development.

The scarcity of books and the fact that I did not have teachers had a similar effect; my deafness deprived me of the opportunity to go to school, though I had to take examinations later on in order to get my certificate. I worked as a teacher of mathematics and physics for nearly 40 years (from 1871). I trained close to 500 young men and 2,000 girls, who graduated from high school. But I myself had had no teachers, save for a limited number of books of questionable quality, and I can be considered a self-educated man.

I got so used to working on my own that when I read textbooks it was easier for me to prove a theorem than to look up the solution. Only I sometimes failed in it.

At the age of 23–24, when I was already a teacher, I presented a few of my papers to the St. Petersburg Society of Physics and Chemistry. These were: "The Theory of Gases," "The Mechanics of the Animal Body," and "The Duration of Stellar Radiation." These works were out of date, as the subjects had already been developed by others before.

Nevertheless, the Society treated me with great consideration, and this encouraged me. Perhaps they have forgotten me, but I still remember Borgman, Mendeleev, Van der Vilet, Bobylev, and especially Sechenov.

When I was 25–28, I tried to improve steam-engines. I had a metal-made and even a wooden (the cylinder was really of wood) steam-engine; both of them were poorly built, but they worked. I also made some fairly good air fans and various pumps. I did not sell these, but just made them as an experiment, and for use in soldering and forging.

After a few years I abandoned these efforts because I saw how ineffective I was technologically when it came to putting my ideas into practice; so in 1885, at the age of 28, I definitely decided to devote myself to aeronautics and the theoretical design of a metal-made dirigible. I worked at it for two years almost continuously. I had always been a dedicated teacher and used to come home from school exhausted, having spent most of my energy there. Only in the evening could I make my calculations and

* In 1882 I taught at the county school in the town of Borovsk of the Kaluga Province; from 1892 on I taught at the Women's Diocesan School in Kaluga.
conduct my experiments. What could I do? I did not have enough time, and I spent all my energy on my students. So I decided to get up at dawn; and after having worked on my treatise, I would go to school.

After two years of such strenuous work I started getting headaches. In spite of everything, in the spring of 1887, I made a first public report about a metal-made dirigible at the Moscow Society of Amateur Scientists. I was treated kindly, with understanding, especially by Yakov Ignatevich Veinberg. Some objections were made, but they were easy to answer. More serious objections could have been made, but they were not raised because the subject was unfamiliar and my manuscript was not there. The manuscript contained 120 sheets (480 pages) and 800 formulas (I still have it). Professor Stoletov had handed it over to Professor Zhukovskii for review. I did not consider my work complete and even asked not to have it reviewed; rather, I requested to be transferred to Moscow.

I was promised this, but for some reason the transfer was never effected. I got sick, I lost my voice, fire destroyed my small library and my models; but the manuscript was then in the hands of Prof. Zhukovskii, and I still have it now. It is called "A Theory of Dirigibles." After a year, I recovered a little and resumed my work.

In the autumn of 1890 I sent new material to the Imperial Russian Society of Technology through the assistance of D. I. Mendeleev. The paper was entitled "Concerning the Feasibility of a Metal-Made Dirigible". Together with this, I sent a model of the dirigible, which could be folded down to a length of 1 arshin [28 inches]. Soon I learnt from the press that the Society had found my calculations and ideas to be correct. Then I received a copy of the remarks of the VIIth Department of the Society of Technology, which gratified me greatly.

A study of winged flight showed me that this type of flight requires far more energy than it had seemed to me at first, and this was subsequently confirmed in practice. As a result I was once again drawn to the dirigible. I remember that after extremely intensive research I wrote a new paper, "The Metal-Made Dirigible." One of my brothers and some acquaintances helped me publish it in 1892. I do not recall greater joy than that which I felt when I received (when I was in Kaluga) the proofs of the paper.

In 1894, I did a last piece of work on aeroplanes, by publishing in the magazine "Nauka i Zhizn'" (Science and Life) my theoretical paper "The Aeroplane"; but even in this work I pointed out the advantages of gas-filled [lighter-than-air] metal airships.

Debates about the dirigible and the aeroplane again induced me to conduct experiments on air resistance. G. Pomortsev and other theoreticians considered the air resistance of dirigibles to be extremely high. My experiments showed that it was not so high and that the coefficient of drag decreased with increasing velocity of the dirigible. I conducted my experiments sometimes in the room, sometimes on the roof when there was a strong wind. I remember how happy I was when I discovered that the coefficient of drag was low in high wind; I almost fell off the roof with excitement.

The sympathetic treatment my work received in the press prompted donations from various sources for the development of aeronautics. Altogether I received 55 rubles which I spent on new experiments on air resistance. I accepted this money reluctantly and with a sense of frustration, as some people did not understand Golubitskii's article about my work, published in
"Kaluzhskii Vestnik" (The Kaluga Herald) (1897), and donated merely out of charity. I even got sick, but withstood it all, hoping to continue my work. But alas, in spite of all the publicity in the newspapers, the sum turned out to be pitifully small. Thus Petersburg sent 4 rubles; but I consoled myself with the fact that the contribution was at least intended for the advancement of aeronautics. At any rate, I am grateful to the Society. Much became clearer to me after the experiments I had conducted and which I described, together with the device I had built, in "Vestnik Opytnoi Fiziki" (Journal of Experimental Physics, ) in an article entitled "The Air Pressure on Surfaces in an Artificial Air Stream" (1899). I presented this work to the Imperial Academy of Sciences. Academician Rykachev reviewed it favorably, thanks to which the Academy granted my request for 470 rubles for the continuation of my experiments.

After a year and a half I sent a detailed report of my experiments to the Academy. It consisted of 80 sheets of text and 58 tables and diagrams. An extract from this report was later published under the title, "Air Resistance and Aeronautics." After this I continued my experiments for some time. They involved various calculations, and they gradually made clearer to me the true nature of air resistance. Each effort brought me closer to the truth, but I never quite achieved the absolute certainty of knowledge.
I would like to pursue my efforts in search of the truth, but where can I get the strength, the means, and the necessary support? My experiments enabled me to derive many new results; but scientists treat new results with scepticism. My findings could be confirmed by anyone else conducting the same experiments, but when will this be?

14   It is difficult to work all on your own for many years, in adverse conditions, without a gleam of hope, without any help. It is true, sometimes I was treated with understanding. For example, in Kaluga, a team of engineers recognized my project for an airship to be practicable. Shouldn't this inspire hope that it will be recognized as such by all thinking and knowledgeable people? For then its realization would not be far away.

FIGURE 5. K. E. Tsiolkovskii with his grandson in 1928

In the twenties, I stopped teaching because of ill health. Even though I used to speak more than listen at lectures, even though I did not like examinations, I still loved teaching. But it absorbed all my strength.
and left me very little energy for my own studies and projects. I wrote, made calculations, and built mostly on holidays and during the vacation.

I worked out certain aspects of the problem of going into space by means of a reaction-driven device such as a rocket. Mathematical calculations, based on scientific knowledge and repeatedly checked, show that it should be possible to go into space with such devices, and perhaps to set up living facilities beyond the atmosphere of the earth. It is likely to be hundreds of years before this is achieved and man spreads out not only over the face of the earth, but over the whole universe.

Almost all of the sun's energy is today wasted for man. (The earth receives half a billionth of the total radiation of the sun.) Is it far-fetched to conceive of utilizing this energy? Or of utilizing the boundless space surrounding the earth? In any case there is nothing wrong with elaborating such ideas, as long as they are the result of serious work.

My partial deafness from childhood has left me completely inexperienced in matters of everyday life and without any contacts. Maybe this is why, even at the age of 68, I have not accomplished much and have not had any notable success.

My whole life has been one of contemplation, calculations, practical work (I even acquired two hernias at it), and experiments. I always had a workshop at home. If it was destroyed, as in a fire or in a flood, I would rebuild and enlarge it.

It is idle to speak of oneself and the trivialities of life while there are so many problems to be solved, papers to be finished or published. My main task is still ahead. Will I have enough strength, will I be able to complete my projects? *

Konstantin Tsiolkovskii

* We may note that Tsiolkovskii's household counts 11 people: his wife, aged 72; an unmarried daughter, aged 48; another daughter with 6 children; and the orphaned child of a third, deceased, daughter.
WRITINGS BY K. E. TSIOLKOVSKII


Kak predokranit' nezhnye veshchi ot tolichkov (How to Protect Delicate Objects from Bumps). — Trudy Obshchestva Lyubitelei Estestvoznaniiya. Moskva. 1891.

Aerostat metallicheskii, upravlyayemyi (Controlled Metallic Aerostat). Moskva. 1892.


Vozmozhno li metallicheskii aerostat (Is a Metallic Aerostat Possible?). — Nauka i Zhizn', No. 51–52. Moskva. 1893.


Grezy o zemle i nebe (Reflections on Earth and Heaven); Effekty vsemirnogo tyagoteniya (The Effects of Universal Gravitation). Moskva, Izd. A. Goncharova. 1895. (See also "Bez tyazhesti." 1914.)

16 Mozhet li Zemlya zavavit' zhitelyam inykh planet o sushchestvovanii na nei razumnykh sushchestvax (Will the Earth Someday be Able to Inform the Inhabitants of Other Planets of the Existence on It of Intelligent Beings?). — Kaluzhskii Vestnik, No. 68. Kaluga. 1896.


Samostoyatel'noe gorizontal'noe dvizhenie upravlyayemogo aerostata (Independent Horizontal Movement of a Controlled Aerostat). — Vestnik Opytnoi Fiziki, Odessa. 1898.

Prostoe uchenie o vozdushnom korable (Simple Instructions on Airships). — Osvetlentsevstupnaya Tekhnika. Moskva. 1898.


Metallicheskii meshok, izmenyayushchii svoi ob' em i formy v primenenii k upravlyaemomu aerostatu i drugim telam (Metallic Shell Changing its Volume and Shape in Adapting to a Controlled Aerostat and Other Bodies). — Sankt—Peterburg. Vseimirnoe Tekhnicheskoе Obozrenie, No. 3. (Otd. ottisk). Kaluga. 1910.


Pervaya model' chisto metallicheskogo aeronata iz volnistogo zheleza (First Model of a Pure Metallic Aeronaut of Corrugated Iron). Kaluga. 1913.

Issledovanie mirovykh prostranstv reaktivnymi priborami (The Exploration of Space with Rocket-Propelled Devices) (supplement to Parts I and II of the work of the same title). Kaluga. 1914.

Prosteishii proekt chisto metallicheskogo aeronata iz volnistogo zheleza (Elementary Design for an All-Metal Dirigible of Corrugated Iron). Kaluga. 1914.

Pervaya model' chisto metallicheskogo aeronata (First Model of an All-Metal Dirigible). Kaluga. 1914.


Nirvana (Nirvana). Kaluga. 1914.


Tablitsa dirizhablei iz volnistogo metalla (Table of Airships Made of Corrugated Iron). Kaluga. 1915.


Obshchi alfavit i yazykh (Alphabet and Language). Kaluga. 1915.

Znanie i ego rasprostranenie (Knowledge and its Propagation). Kaluga. 1915.

Gore i genii (Agony and Genius). Kaluga. 1916.


Gondola metallicheskogo dirizhablya i organy ego upravleniya (Gondola of a Metallic Airship and its Control Devices). Kaluga. 1918.

17 Vne Zemli (Beyond the Earth). Science-fiction story. — Priroda i Lyudi, Nos. 2–14. 1918. (In the foreword to this story the author mentioned that the first chapters were written in 1896.)


Bogatstva vseleinnoi (The Resources of the Universe). Kaluga. 1920.

Vne Zemli (Beyond the Earth). Kaluga. 1920.

Istoriya moego dirizhablya (History of My Airship). Kaluga. 1924.


Chetyre sposoba nosit'sya nad sushei i vodoi (Four Methods of Floating over Land and Water). — Vozdukhoplavanie, Nos. 6–7:10. Kaluga. 1924.


Issledovanie mirovykh prostranstv reaktivnymi priborami (The Exploration of Space with Rocket-Propelled Devices). (Reprint of the works of 1903 and 1911 with some alterations and additions.) Kaluga. 1926.


Obshchechelovecheskaya azbuka, pravopisanie i yazyk (Universal Alphabet, Spelling and Language). 1927.


Dopolnenie k "Obrazovaniyu solnechnykh sistem" (Supplement to "Formation of Solar Systems"). Kaluga. 1928.


* A reference to this work can be found in Z.Fluchtech. Motorluftschiff., p.482. 1927.
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Um i strasti (The Mind and Passion). Kaluga. 1928.
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Ot samoleta k zvezdoletu (From Airplane to Starplane). — Iskry Nauki, No. 2:5. Moskva. 1931.

18 Besides the cited works of K. E. Tsiolkovskii other small works were also published in journals such as: V Masterskoi Prirody, Tekhnika i Zhizni, Vozdushoplavanie, Syvag', Ogonek, Biologiya i Zhizni, O Dirizhable, etc.

MANUSCRIPTS BY K. E. TSIOLKOVSKII

For an index of manuscripts by K. E. Tsiolkovskii see the books: "Istoriya moego dirizhablya (The History of My Airships), p. 16. Kaluga. 1924;
1883  Teoriya gaza (Gas Theory)
Podobnye organizmy (Similar Organisms)
Izmeneniya sily tyazhesti (Change in Force of Gravity)
1886  Teoriya aerostata (Theory of the Aerostat)
1890  K voprosu o letanii posredstvom kryl'ev (Flying with Wings)
O vozmozhnosti postroeniya metalliceskogo aerostata (Possibilities
of Constructing a Metallic Aerostat)
1893  Podobnye Organizmy (Similar Organisms)
1894  Svobodnoe ot tyazhesti prostranstvo (Space Free of Gravity)
1896  Na planetakh (On the Planets)
1901  Otchet dlya Akademii Nauk o sdelannykh mnoyu opytakh po sopro-
tivleniyu vozduka (Account to the Academy of Sciences on My
Experiments with Air Resistance)
1903  Estestvennye osnovy nравственности (The Natural Basis of Morality)
1908  Aerostat i aeroplan (prodelzhenie naapechatannoi raboty pod takim zhe
rasvaniem (Aerostat and Airplane (continuation of work published
under this title))
1915  Vydelenie cheloveka iz tsarstva zhivotnykh (The Division of Man
from the Animal Kingdom)
1916  Razvitie dushi i tela (Development of Soul and Body)
Svobodnoe ot tyazhesti prostranstvo (Space Free of Gravity)
Obrazovanie prosteishikh zhivykh sushchestv (The Formation
of Simple Living Beings)
Usloviya zhizni v inykh mirakh (Conditions of Life in Other Worlds)
1917  Razgovory o metallicheskom dirizhable (Conversations on a Metallic
Dirigible)
Ideal'nyi stroi zhizni (The Ideal System of Life)
1918  Mekhanika i biologiya (Mechanics and Biology)
Svoistva cheloveka (The Characteristics of Man)
Sovremennye obshchestvennye ustanovleniya (The Modern Social
Establishment)
Kak ustroit' obshchestvo i sozdat' blagosostoyanie (How to Build
Society and Create Prosperity)
Obshchestvennyi stroi (The Social System)
1918  Priklyuchenie atoma (Adventure of the Atom (121 p.))
1919  Pervoprichina (The Source)
Na Veste, Mysl', Obshchestvennye ustanovleniya (On Vesta (5 p.),
Thought (5 p.), Social Establishment (33 p.))
Nachalo rastenii na zemnom share (Beginning of Growth in the World
(46 p.))
1920  Puteshestvie zemli i solntsa (Travels of the Earth and Sun (29 p.))
Vooobrazhenie (The Imagination (10 p.))
1921  Mirovye katostrofy (World Disaster (30 p.))
Iz proshlogo zemli (From the Past Earth (6 p.))
1923  Energiya solnechnogo lucheispuskaniya (po otnosheniyu k dvizheniyu
nebesnykh tel) (Power of Solar Radiation (in relation to the
movement of celestial bodies) (28 p., 6 figures))
1924  Voda v bezhvodnych i bezoblachnykh pustynyah (Water in Arid and
Cloudless Deserts (8 p. and 1 figure))
Pochemy trudno osushchestvlyat' moi dirizhabl' (Why it is Difficult to
Accomplish my Dirigible)
<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
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<tbody>
<tr>
<td>1926</td>
<td>Ustroistvo zhilishch v sukhikh i zharkikh pustynyakh (Building Houses in Dry and Hot Deserts (11 p. and 7 figures))</td>
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<tr>
<td>1926</td>
<td>Prostoi solnechnyi nagrevatel' (Simple Solar Heating)</td>
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<tr>
<td>1927</td>
<td>Galileiskii myslitel' (Gallileo's Thinker)</td>
</tr>
<tr>
<td>1927</td>
<td>Usloviya biologicheskoi zhizni vo vseleinnoi (Biological Conditions of Life in the Universe (9 p.))</td>
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<tr>
<td>1928</td>
<td>Obrazovanie solnechnoi sistemy (vyvody) (Formation of the Solar System (conclusions) (3 p.))</td>
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<tr>
<td>1928</td>
<td>Sovershenstvo vseleinnoi (Perfection of the Universe (7 p.))</td>
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<tr>
<td>1928</td>
<td>Efirnyi ostrov (Ethereal Islands (13 p.))</td>
</tr>
<tr>
<td>1928</td>
<td>Strannyi sluchai v 1885–86 godu (Strange Events in 1885–1886 (2 p.))</td>
</tr>
<tr>
<td>1928</td>
<td>Teoremy zhizni. Sluzhat poyasneniem k Monizmu i Etike (Theorems of Life. Serve as an Elucidation to Monism and Ethics (12 p.))</td>
</tr>
<tr>
<td>1929</td>
<td>Chto delat' na zemle (What to Do on Earth (27 p.))</td>
</tr>
<tr>
<td>1929</td>
<td>Glavnye vyvody iz &quot;Novyi aeroplan&quot; (Main Conclusions from &quot;The New Airplane&quot; (4 p.))</td>
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<tr>
<td>1929</td>
<td>Dostupny li planety (otvet) (Are the Planets Accessible? (reply) (3 p.))</td>
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<tr>
<td>1929</td>
<td>Kartina vseleinnoi (Picture of the Universe (28 p.))</td>
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<tr>
<td>1929</td>
<td>Obshchestvennoe ustroistvo (otryvok, 9 str.) (Social System excerpts, 9 p.)</td>
</tr>
<tr>
<td>1929</td>
<td>Reaktivnyi aeroplan (The Reaction-Driven Airplane (11 p.))</td>
</tr>
<tr>
<td>1929</td>
<td>Trudy o kosmicheskoi rakete (Studies on a Space Rocket (8 p.))</td>
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<tr>
<td>1929</td>
<td>Rukovoditeli (Leaders (11 p.))</td>
</tr>
<tr>
<td>1929</td>
<td>Zvezdoplavanie (Astronautics (3 p.))</td>
</tr>
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**Chapter II**

**SURVEY OF TSIOLKOVSKII’S PRINCIPAL WORKS**

20 We do not propose to give here an account of all of Tsiolkovskii’s works, or even of an appreciable number of them. We will only present a broad outline of some of his works on aerodynamics, aeronautics and cosmography. The foregoing list of his works shows the versatility of Tsiolkovskii’s scientific researches. The reader interested in individual papers of his may refer to the sources cited in the list. Many of the papers are now hard to come by, however, and the publication of his collected works would therefore be quite desirable.

We present Tsiolkovskii’s theory of rocket-propelled space flight in some detail, as he is credited with being the first ever (not only in Russia) to have put the theory of rocket space flight on a scientific basis.

**WORKS ON AERODYNAMICS**

Tsiolkovskii began his experiments on air resistance at a time (1891) when very little was being done on the subject (by D. I. Mendeleev and Academician Rykachev, among others). His work subsequently came to the attention of certain people through the press, and they donated 55 rubles to help him pursue his experiments. In the course of time the Academy of Sciences also allocated 470 rubles, with which Tsiolkovskii conducted another series of experiments in 1900.

In 1899 Tsiolkovskii had a small booklet (32 pages) published in Odessa, which bore the title "Air Pressure on Surfaces in an Artificial Air Stream." This was an offprint from the journal "Vestnik opytnoi fiziki i elementarnoi matematiki" (Journal of Experimental Physics and Elementary Mathematics).

In this valuable paper Tsiolkovskii derived, on the basis of his experimental findings, the fundamental laws of air pressure exerted on bodies of various shapes.

For his experiments and with the meager means at his disposal, he managed to build a wind tunnel, the first in Russia, mounted a balance in it and measured the drag exerted on various bodies placed in the air stream.

21 Tsiolkovskii’s setup included the following components (see drawing): The blower B, driven by weight W (from 1/2 to 16 pounds); its nozzle cross section was about 1,225 cm² (35×35 cm), and the air velocity could reach 5.15 m/sec. A smooth air flow was obtained by means of a grating, G, 25 cm deep (in the direction of flow) set in the discharge duct. The test model M was placed on a support floating in a water-filled vessel V resting on stand S. Under the air pressure the support pulled on drawstring D and produced a bending force on flexible pointer P rigidly fixed to the stand.
The deflection of the pointer was read off scale R. The scale was calibrated before the experiments by means of a system of pulleys and weights.

The air-stream velocity was determined from the pressure exerted on square plates set perpendicular to the flow.

The tests involved: 1) flat plates perpendicular to the flow; 2) flat plates at an angle to the flow; 3) plates with different length-to-width ratios; 4) parallelepipeds; 5) cylinders; 6) polyhedrons; 7) spheres; 8) bodies of revolution; 9) half-cylinders; 10) half-spheres; 11) cones; 12) a model of Schwartz's balloon. In addition the coefficient of air friction was also determined.

It is really astonishing that, given his working conditions, Tsiolkovskii was able to accomplish such extensive, thorough work and to formulate the basic laws of air resistance with a fair degree of accuracy.

He observed, in particular, that the resistance of a flat plate varied with its elongation.

In 1903, after receiving a grant of 470 rubles from the Academy of Sciences, Tsiolkovskii resumed his experimentation with fresh vigor and built another blower producing an air stream 1 sq. arshin in cross section. He published his experimental findings the same year, in the article "Air Resistance and Aeronautics." He performed experiments with rectangular plates of various elongations set perpendicular and at an angle to the flow, with cylinders, curved cylindrical plates, parallelepipeds, surfaces formed by the revolution of a circular arc about its chord, and others.

He carried out hundreds of experiments with a large number of models, in which he also determined the coefficients of lift and drag.

In 1908 he published in the journal "Vozdushoplatavet" (The Aeronaut)* the results of new experiments made on flat plates of various configurations and on curved surfaces; concerning the latter he noted that wings should be shaped like long triangles more strongly cambered at the front.

In 1930 he put out a new theoretical study, "The Pressure on a Plane Moving along a Normal in Air."

* No.8, p.277.
THE METAL AIRSHIP

As early as 1890 Tsiolkovskii drew up a project for a metallic airship. On the advice of D. I. Mendeleev, the project was discussed in 1893 at the 7th Department of the Russian Technical Society, but it never got beyond discussion. The construction of a metal airship was in fact attempted in various countries at one time or another (e.g. Schwartz's dirigible, Epson Slate's airship now being built in America, etc.)

Schwartz's ideas on the subject are, however, worth considering; first, he proposed a metal airship at a time when in Russia no one had ever thought of it; and second, because his concept, unlike other projects, was based on the use of a flexible hull of corrugated metal.

This flexibility makes it possible to maintain constant lift by varying the volume of the airship.

In 1892 Tsiolkovskii's first paper on his airship, "The Metal Dirigible," was published in Moscow.

Here are some excerpts from this work, which explain the basic design.

1. The airship envelope consists of a flexible corrugated-metal hull, elongated along its horizontal axis and capable of freely changing its volume within certain limits. The corrugations run transversely to the length. The term "free change of volume" is meant to denote contraction and expansion of the hull without stress, though this is, strictly speaking, impossible.

2. The temperature of the gas in the envelope, and thus also the volume of the envelope, is controlled by means of pipes heated by the combustion products from the motors.

3. Given in the paper are also the design data calculated for two airships, 7,312 and 58,500 m³ in volume, as listed below. The airships are supposed to be driven by oil engines.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Small airship</th>
<th>Large airship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, m</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>Length, m</td>
<td>98</td>
<td>196</td>
</tr>
<tr>
<td>Volume, m³</td>
<td>7,312</td>
<td>58,500</td>
</tr>
<tr>
<td>Engine horsepower</td>
<td>16</td>
<td>127</td>
</tr>
<tr>
<td>Velocity, km/hr</td>
<td>34</td>
<td>43</td>
</tr>
<tr>
<td>Passengers</td>
<td>12</td>
<td>100</td>
</tr>
</tbody>
</table>

In 1893 Tsiolkovskii published the second part of his book on the same topic.

* [This refers to the time of writing of this book in Russia, i.e. just at the beginning of the thirties.]

** [Historical note: the major projects for metal airships and balloons were as follows:]

In 1670 Francesco Lana designed an airship which was to be kept aloft by four copper globes from which the air was evacuated.

In 1831 Dupuits-Delecosse and Mongeaux built an air balloon 10 m in diameter out of sheet copper. The venture was unsuccessful.

In 1866 some attempts to build a metal air balloon were made by Beiman, and in 1877 by Micelio-Picasso.

In 1891 Bosse tried to build a steel-made balloon filled with rarefied air.

In 1893 similar attempts at building a metal balloon were made by Schwartz.

In 1897 — Fontana.

In 1898—1900 — Hadron, Sibbelot, Hidt, Russler [Wrestler?].
In this book Tsiolkovskii describes the experiments he performed to determine the structural stability of the dirigible. He proposed, for instance, making a model of the envelope out of some impermeable material, blowing it up with air, then attaching a weight to it and immersing it in water. He also tested the cross-sectional deformation of the airship by shaping ribs from pieces of wire and suspending weights at various points on them.

Further he experimented with a hull which had a pair of girders running along its top and bottom; by means of tension cables stretched between them, these flexible bands could be drawn together, thus altering the shape and volume of the envelope.

Figure 7 depicts the longitudinal and transverse section of Tsiolkovskii's small airship (of 6,000 m³ volume). It is 64 m long, 17 m high and 12.8 m wide.

**FIGURE 7. Tsiolkovskii's small metal airship**

The side view shows: 1 – upper contour of the hull; 2 – upper girder (band), to which are fastened the cables holding the nacelle; 3 – lower girder (band). The airship is powered by gas, gasoline or oil engines and has a speed of 32 km/hr.

The hull is made of transversely corrugated sheet metal. There are no corrugations where the hull meets bands 2 and 3. Below the envelope is a set of wheels whose purpose is to keep the cables pulling the bands together under an even tension. The floor of the nacelle is also made up of a flexible band, and the whole nacelle can bend slightly when the tension cables are drawn tight. A propeller is mounted at each end of the nacelle. For steering there can be either rudders or lateral propellers. Horizontal stabilization is achieved by means of a longitudinal tube through which liquid can be passed from one end to the other, but this device may be replaced by a weighted trolley. The gas in the envelope is heated by the combustion products running through pipes.

Tsiolkovskii's book contains a table listing the specifications of airships of his design, with volumes ranging from 890 to 1,566,000 m³.

In 1904 Tsiolkovskii gave in his book "A Simple Theory of Airships and their Construction" the plan and description of a metal airship for 200 passengers (Figure 8), with a volume of 138,400 m³. The design is on the whole similar to the one presented above.

In 1914 Tsiolkovskii included in his book "Elementary Design for an All-Metal Dirigible of Corrugated Iron" a number of drawings of his airship with details. These drawings were later reproduced in his article "The Steel Airship" which appeared in the journal "Aviatsiya i Khimiya".
We quote his article here, as it gives a concise and yet fairly complete account of the construction. "My design for a metal airship is clear from the attached drawings. The hull, whose longitudinal and transverse sections are given in Figure 9 (with the cable linkage shown inside), consists essentially of the following parts (Figure 10a): upper keel (1), hinge joints enclosed in slit tubing (2), corrugated steel sides (3), lower keel (4) and end-plates (5). Drawings b, c and d show schematically the cross section of the hull at different degrees of inflation; the hinge-joint details are given in Figure 11a.

"The longitudinal cross section of the whole airship, including the nacelle, is schematically represented in Figure 11b. In its lower part are pipes (black lines) for heating the gas (old design), winches for the pulley cables, the nacelle, rudders, motors with propellers, and a row of viewports.

"The temperature of the filling gas is controlled by the combustion products discharged from the motors (indicated in Figure 11c as a flame burner). A valve enables them to be directed either into the heating pipes or to the outside. This makes it possible to vary the lifting power and to balance the airship against meteorological effects.

"This kind of airship can change in volume, it retains its shape and allows the lift to be varied without any loss of gas or ballast. The design does not properly belong to either the rigid, the non-rigid or the semi-rigid type of construction.

"The given structure owes its strength and light weight to the fact that the hull is supported by the gas and all the components are subject to tensile forces only. It may be built out of galvanized or lead-plated iron, chrome steel, or some other metal."

In his later works Tsiolkovskii further elaborated his ideas on metallic airships; he calculated their flight characteristics, stability, navigability, size, etc.

Among these, in his paper "Table of Airships Made of Corrugated Iron" (Kaluga, 1915), p. 6, he gives new data for 24 airships of various sizes, ranging from 12 to 1,800 m in length, with volumes from 16,275 to 54,900,000 m³ (7/4 capacity), carrying from 0 to 131,800 passengers.
In the book "Design for a 40-Passenger Metal Airship," published in 1930, Tsiolkovskii gives the plans for a metal-made airship 20 m high, 120 m long and 23,576 m$^3$ in volume, but he points out that the specified dimensions are actually inadequate and that the height, for instance, ought to be made as large as 100 m. The basic design remains essentially the same. The airship has an elongation* of 4 when flat, and 6 when inflated. Its lifting power is 27 tons, filled with hydrogen to full capacity, and 21 tons filled to three-quarter capacity. The lift can be varied by heating the gas. Given a flying speed of 78 km/hr the engine power is $2 \times 132 = 264$ hp.

* [The ratio of length to maximum width, known as the fineness ratio of the airship.]
Private institutions and the government occasionally allocated some funds to help Tsiolkovskii carry on his experiments with airship models. Due to the lack of systematic material support, however, he was never able to test whether a full-scale airship could be realized in practice. In his book "The Corrugated-Steel Airship" (1929), Tsiolkovskii summarizes his ideas on the construction and operation of his airship.

THE DIRIGIBLE AND THE AEROPLANE

This work of Tsiolkovskii's appeared at a time when little had been written on the subject of aeronautics and aviation. The treatise was published in the journal "Vozdushopolavatel'" (The Aeronaut) during 1905–1908, and covered 260 pages of formulas, tables, reports of the author's experiments, etc.; it constitutes a major independent study on lighter- and heavier-than-air flight.

We highlight here only the main points treated in the work:

1. Conditions of equilibrium for a balloon in vertical flight. In this part the author develops the theory of ascent of a free balloon and considers the effect of the temperature and pressure, the variation of the balloon volume and the effect of the degree of filling. He formulates the idea of a metal-made dirigible and discusses the shape it should have.

2. Further he proceeds to describe the metal airship of his design. He gives drawings of its construction, analyzes its shape and the deformations that can be produced, breaks down the envelope into plane sections, calculates its strength (corrugated surface), estimates the stability of the craft, examines in detail how the shape of the hull can be modified by a linkage of tension cables, and determines the weight of the airship. (The treatment of airships takes up 221 pages.)

3. The last part deals with air resistance to the motion of bodies of various shape, wings, aeroplanes and airships (39 pages).

THE AEROPLANE OR THE AVIFORM (HEAVIER-THAN-AIR) FLYING MACHINE

This was the title of Tsiolkovskii's paper (46 pages) which appeared in Moscow in 1895; at that time no detailed studies of aeroplanes had been made in Russia. In his work Tsiolkovskii evolved an original theory of flight of the aeroplane, described its structure and calculated the engine power. Figure 12 shows schematically the aeroplane Tsiolkovskii designed in 1895.

THE NEW AEROPLANE

In 1929 Tsiolkovskii published a new work entitled "The New Aeroplane," in which he proposed the construction of large aircraft of the flying-wing type.
Here is how he conceives his device (Figure 13):

Imagine a spindle-shaped envelope of circular cross section, tightly inflated with air or oxygen, with a diameter of at least 2 meters and a length of at least 20 meters.

A parallel row of such spindles joined side to side forms a fluted slab with spikes at the front and rear. Each tip carries an air-screw driven by a motor. At the rear sides there are elevator vanes and on top there are rudder vanes.

For takeoff the aeroplane is set on floats or wheels which drop off in flight. It may land in water, or else on a flat field with snow.

According to Tsiolkovskii this aircraft has the advantage of being lightweight, free from protruding parts, sturdy due to the internal superpressure, of simple construction, able to fly in thin atmosphere, etc.
THE REACTION-DRIVEN RAILCAR

In 1927 Tsiolkovskii proposed a design for a high-speed wheelless train. Figure 14 shows the top and front views of a railcar of this type. On the bottom B of the car there are semicircular channels C–C. The track bed T lies level with the rails R. Independently running motors blow into the channels compressed air which spreads out in the narrow space between the car and the track, lifting the car a few millimeters, and escapes around the edges of the car base. The car is thus held aloft by a thin cushion of air and encounters in its motion only an insignificant amount of air friction. The base has flanged edges which prevent the car from slipping off the track, and which also reduce the leakage of air by causing the flow to be sharply deflected. The car bottom is finely grooved, in order to retard the outflow. Air is drawn in through the front opening of the car, part of it is diverted through peripheral vents [to the bottom] and part is expelled through the rear end, where it exerts a pressure by reaction and drives the car forward. The front and rear of the car are tapered in order to lower the air resistance. The motor feeding the rear air passage may be separate from the other motors. Given in the figure is also the cross section of another form of railcar, with a downward curving bottom and without flanged edges. This design provides higher stability, but the track is then more difficult to build.

Figure 15 gives a visual impression of Tsiolkovskii's railcar, seen in transit over a river (drawing by I. Gusev).
THE FLYING MECHANISM OF INSECTS AND BIRDS AND THEIR MODES OF FLIGHT

In this paper, printed in the journal "Tekhnika vozdukhoplavaniya" (Aeronautical Engineering) (1911, No. 3), Tsiolkovskii examines the flight of insects and birds in still and in moving air; in the latter case he takes into consideration both steady and unsteady air flow.

THE KINETIC THEORY OF LIGHT

In the "Density of the Ether and its Properties" (39 pages) Tsiolkovskii gives a method for determining the ether density and its properties on the basis of the kinetic theory. He treats the ether as a very tenuous expansible gas, and it is this hypothesis that leads him to the idea of determining its density. From the value obtained for the density he calculates the solar constant and finds it to be in agreement with experimental results. He then proceeds to determine the pressure of the ether, and finally formulates assumptions concerning the distribution of the ether throughout space, the structure of matter, the origin and polarization of light, etc.

30 REFLECTIONS ON EARTH AND HEAVEN

One of the earliest books in which Tsiolkovskii envisions the prospects of interplanetary travels is "Reflections on Earth and Heaven and the Effects of Universal Gravitation" (Moscow, 1895). In this engrossing book he discusses the formation of the universe and the laws of universal attraction, describes what would occur if gravity were to disappear on earth, considers ways in which an environment with non-terrestrial gravity could be created on earth, theorizes about the possibility of life without an atmosphere, explores the asteroid belt, determines the energy of the sun, and concludes with an account of the structure of the universe and the influence of gravitation on the generation of heat and light. The book still retains its interest today; it offers the reader a fascinating glimpse into the vast reaches of interstellar space and all its wonders.

Let us examine a few of the many ideas that the author develops throughout the book.

1. A project for a rotating laboratory within which it would be possible to study the change in the weight of bodies and the phenomena produced by the centrifugal force. This would provide the means of determining the excess weight that may be safely sustained by living beings.

2. A laboratory dropped from a height of 300 meters (the Eiffel Tower) into water, for studying phenomena under conditions of weightlessness. Similarly, use could be made of a trolley attached to vertical horseshoe-shaped rails and allowed to coast down one arm and up the other.

3. Experiments with a man immersed in water and dropped from a height.

4. Shooting out of a long cannon a shell containing a man immersed in liquid.
5. A life system placed under space conditions, within a glass enclosure containing animals, plants, soil, carbon dioxide and moisture. The system is supplied with energy by the sun.

6. Living conditions on asteroids of various sizes, and the probable shape and constitution of beings that might inhabit them.

7. Creating artificial "gravity" in a manned compartment in outer space, by making it rotate about a center of gravity with an attached counterweight on chains.

8. Taking off from asteroids into outer space, with the use of solar engines deriving energy from the sun. These engines are constructed as follows. Imagine a metal container which is perfectly airtight but can change its volume, like a bellows. The container is filled with the vapor of some liquid. One half of its outer surface is black, absorbing the sun's rays, and the other half is shiny, reflecting them back. If the black side is turned towards the sun the vapor expands, and if the shiny side faces the sun the vapor contracts. When the container rotates, the vapor will successively expand and contract, thus yielding work. With the aid of such engines, carriages can be made to move along circular rails set on radial pillars at various levels around the asteroid (Figure 16). Due to the low gravity of the asteroid, at a certain level the centrifugal force will counterbalance the weight of the carriage, and higher up it will even cause the carriage to fly off unless it is linked to the rail. If it is disengaged from its rail the carriage will become a satellite either of the asteroid or of the sun, depending on its velocity of motion. In the case of a large asteroid the centrifugal force alone is not sufficient, and the carriage must be given an excess velocity. The carriages may be made to move along parallels or along meridians and can either leave the asteroid themselves or else release objects into space through a hatch.

9. Steering the motion of (small) asteroids, and disintegrating them into parts.

10. Gathering the sun's heat by means of mirrors in space and concentrating it on given spots.
Chapter III

TSIOLKOVSKII'S ROCKETS AND THEIR PROJECTED FLIGHT

"There are countless planets, like many island earths, in the boundless ether ocean. Man occupies one of them. But why could he not avail himself of others and of the might of numberless suns?"

K. Tsiolkovskii

"Execution is preceded by thought, Exact calculation by imagination."

K. Tsiolkovskii

Before discussing Tsiolkovskii's works dealing with the investigation of cosmic space by means of rockets, we present the introductory article he wrote at our request in Kaluga on 14 May 1927 for the exposition that follows.

TSIOLKOVSKII'S INTRODUCTION

The present book provides Russian readers for the first time with a fairly comprehensive account of my rocket, written by an expert. This project is designed to bring my invention to the knowledge of the public at large, and I therefore gladly give the author a free hand to draw on my works and reproduce out of them whatever he deems necessary.

The fate of any idea or discovery is difficult to foresee: whether it would ever be put into practice, and how soon — within decades, or rather within centuries — in what form it would be realized, how it could improve man's life, and whether it might radically change our outlook and our science.

Electricity, for instance, was known thousands of years ago, but its scientific and practical value is recognized only now. Plans for the aeroplane date back to Leonardo da Vinci, and it has only recently become a reality. The dirigible has been relegated to obscurity for more than a hundred years now (though its future is yet to be seen). Some ideas have been lying dormant for as long as thousands of years, such as the concepts of Democritus which came to be revived by Proust and Dalton.

Many inventions have been made, which have found no application to this day. And some, for which high hopes were held, failed. Such has been the
fate, unfortunately, of a great many inventions. Conversely, some developments which at first seemed insignificant later proved to be of tremendous value. For instance, the discovery of the dark lines in the solar spectrum, on which all of astrophysics is based. The use of various types of engines and production machinery has already had a great impact on human welfare, and it has far-reaching consequences. In their initial stage most of these inventions provoked amusement, if not worse.

There is little doubt that the idea of using reaction-driven devices (space rockets) for interplanetary travel is steadily gaining acceptance. Its realization will bring within our reach an unlimited amount of space and the bountiful energy of the sun, which is 2 million million times greater than that received by the earth. Consider the freedom of movement in the ether, the absence of gravity, and many other advantages I have discussed before—or the manifold puzzles that astronomy will finally be able to answer with the knowledge uncovered during spatial voyages.

I believe that the prospect of the inestimable benefits to be derived will challenge man to venture into space and will make him exercise his full creative powers to this end.

This project, which is in the realm of pure speculation as yet, might eventually be implemented as follows: 1) static tests; 2) propulsion of a reaction engine (rigged with instruments) on the ground (an airfield); 3) launching it to a small height and letting it glide to a landing; 4) reaching the upper atmosphere as far as the stratosphere; 5) launching a craft beyond the atmosphere and letting it glide to a landing; 6) setting up orbiting stations outside the atmosphere (like miniature moons near the earth); 7) utilizing the sun's energy for breathing, nourishment, and other everyday purposes; 8) using this energy for traveling throughout the planetary system and for industry; 9) visiting the smallest bodies of the solar system (asteroids and planetoids) in toward the sun and out away from it; 10) establishing bases on these bodies; 11) putting bases on the small moons and minor planets (landing on the major planets is so difficult that I think it is irrelevant at this point); 12) colonizing the entire solar system (we might as well discount for the present migration to other primaries, i.e., other suns).

Some current authorities have committed themselves to the view that interplanetary travel is feasible, and within the near future at that. May one take this as a guarantee of success, then? Unfortunately not, as history tends to show. There have been so many spurious inventions supported in good faith by competent people and so many considered worthless that later proved of great value.

One works and hopes, and only life itself can settle one's questions and doubts. Time will tell. But such a lot of nonsense has been and is still being maintained against space flights, even in educated circles. People naturally oppose the new and unfamiliar, and a whole book would not suffice to dispel the misconceptions prevailing on the subject. For the time being, continued publicity and experimental work will have to do.

The early pioneers in the field were—Kibal'chich, Ganswindt, Hoefft, Ulinsky, Tsiołkovskii, Pelterie, Dietl [?], Weber, Schiller, Hohmann, Hoffmann, Oberth, Valier, Scherschevsky, Vetchinkin, Yashchurzhinskii, Goddard, Jenkins, Lorin, Tsander, Nikol'skii, Lindemann, Wolf, Rynin, and others with whom I am not acquainted.*

* I have underscored the names of those whose work is best known to me.
However, until my own works of 1903, the rocket drive was generally intended for air flight only. It is only thereafter that consideration was given to rockets for flying beyond the atmosphere.

In Russia highly readable popular accounts of the principles of rocketry were written by Perel'man and Ryumin. This work was continued by Davidov, Lapirov-Skoblo, Modestov, Pryanishnikov, Egorov, Manuilov, Babaev, Glushkov, Bokht, Chizhevskii [quoted in Western literature as Tchijevsky also], Alchevskii, Shmurlo, Ryabushinskii, Rodnykh, Redin, Solov'ev, Shirinkin, and many more whom I am unable to name. Some articles are signed with initials; others bear no signature at all. Many public debates and lectures were held on the subject of space rockets (Pryanishnikov, Vetchinkin, Fedorov, and others).

The great merit of these people is to have steadfastly upheld new ideas, until these might be put into practice or else proven ill-founded or totally unrealistic. This is an estimable virtue that very few possess.

Scoffers and sceptics are legion—and much of value has been irretrievably lost at their hands. Men seem to have always been afflicted with a callous tendency to squelch greatness, seeking to destroy that which ultimately was for their own good. Will modern man ever be able to transcend this self-defeating urge of his?

The civilized countries are by now learning to avoid this pitfall. There are still places, though, where fresh ideas are held up to ridicule or just casually dismissed. Criticize, if you will, well-established ways of thought; but the newly born must be helped along until it comes into its own.

K. Tsiolkovskii

TSIOLKOVSKIi'S WRITINGS ON ROCKET SHIPS

The Russian Scientist K. E. Tsiolkovskii firmly believed in the possibility of traveling into space and reaching the planets by means of rockets. His first work, entitled "The Exploration of Space with Rocket-Propelled Devices," was published in 1903 in the journal "Nauchnoe obozrenie" (Scientific Review) (St. Petersburg 1903, ** No. 5, page 45). He later recapitulated his fundamental idea in some other journals and individual papers, together with several additions.

In 1924 this work was reprinted in Kaluga under separate cover, as a booklet, "The Rocket in Space."

Tsiolkovskii's concluding work, which presents his thoughts on the interplanetary travels likely to take place in the future, was published in Kaluga in 1920 as a booklet entitled "Beyond This Life." It describes in fictional form rocket flights in 2017 and living conditions in space around the earth.

Here is Tsiolkovskii's own account of the development of his ideas on rockets.†

- The idea of using rockets for interplanetary flight dates from long before Tsiolkovskii's time (e.g. in the writings of Cyrano de Bergerac). Credit goes to Tsiolkovskii for being the first to have developed the idea on a scientific basis (cf. our books "Dreams, Legends and Early Fantasies" and "Rockets"). [English translations by IPST, TT 70-50111. 1970; TT 70-50114. 1971.1]. N. R.

** In the article which appeared under that title in "Vestnik Vozdushnoi Plavani" (Aeronautical Journal), 1911, No. 19, page 16, Tsiolkovskii mentions that he had derived the final flight equations for a reaction engine as early as 25 August 1898 and that he had been dealing with the subject already before that.

† Ibid, page 16.
"For a long time I regarded rockets, in the same way as others did, as diversions of limited practical use. I remember well how the thought struck me of making calculations for rockets.

I think the first seeds were sown by the imaginative tales of Jules Verne, which stimulated my mind. I was assailed by a sense of longing, and this set me to thinking in a specific way. Of course reasoning alone would have led nowhere without recourse to science.

It also seems to me, though I could be wrong, that the basic drive to reach out for the sun, to shed the bonds of gravity, has been with me ever since my infancy. Anyway I distinctly recall that my favorite dream in very early childhood, before I ever read books, was a dim consciousness of a realm devoid of gravity where one could move unhindered anywhere, freer than a bird in flight. What gave rise to these yearnings I cannot say; I never came across stories like that, but I dimly perceived and longed after such a place unfettered by gravitation.

An old page in my manuscripts with the final formulas for a reaction-driven device is dated 25 August 1898. I evidently must have been working on the subject earlier, and it was not the mere flight of a rocket that fascinated me but the exact calculations.

Be it far from me to claim to have solved the problem. The course of events necessarily is: thought, imagination, description, these are followed by scientific calculation, and then, finally, execution completes the thought.

My own work concerns the middle phase of the process. I know better than anyone the chasm that separates an idea from its realization — I have made many calculation, and also built a lot with my own hands.

But the idea must be there; for execution is preceded by thought, and exact calculation, by imagination.

I shall be happy if my work induces others to further efforts.

PROSPECTS OF ROCKET FLIGHTS

In many of his papers Tsiolkovsky describes how he envisions future space flights using rockets and discusses their implications.

"First it might be possible to circumnavigate the earth in a rocket; then some trajectory could be followed about the sun to reach any of the planets, or the rocket could approach the sun or escape it altogether like a comet, wandering for thousands of years in the interstellar void until it comes close to a star, which would become a new sun to the space travelers or their descendants.

The next stage might be to set up interplanetary bases around the sun, utilizing for this purpose the asteroids floating about in space.

When the sun has exhausted its energy it would be logical to leave it and look for another, newly kindled, luminary still in its prime. This may possibly be done earlier, by adventurous souls seeking fresh worlds to conquer.

Man may thus gradually push outward — for there would be nothing much to do on the sun, even after it becomes covered with a cool crust. There would be little point in going to the heavy planets, too, save for research
perhaps; they are difficult to reach, and staying on them would entail being shackled down by a crushing gravity, living in cramped quarters and leading a dismal confined existence. The planet is the cradle of intelligence, but one cannot stay in the cradle forever.

There was a time, not so long ago, when the possibility of determining the physical properties of heavenly bodies would have been considered a wild notion even by eminent scientists and thinkers. That time is now past.

I think that today the prospect of directly exploring the universe might appear a still more radical idea. Setting foot on the asteroids, picking up a stone from the lunar soil, building stations that float in space, living in orbit around the earth, moon and sun, observing Mars from a distance of a few tens of versts,\(^7\) landing on its satellites or on its surface — all this may well seem highly fanciful. But the use of reaction-propelled craft gives astronomy a stepping-stone to the sky and opens up exciting new vistas. Are we not deterred by the magnitude of the force of gravity more than we should really be?

Advanced societies tend to live nowadays in a more and more artificial setting — and is this not what progress means? Protection against the weather, against temperature extremes, overcoming gravity, controlling wild animals and pests, destroying bacteria — all this helps in creating an artificial environment for man.

Out in space this artificiality will be developed to the utmost, but this will only produce the most favorable living conditions for man.

In the course of time the new conditions will engender a new breed of beings, and the artificiality surrounding them will be reduced and maybe even gradually eliminated. As in the past aquatic creatures crawled onto the land and little by little turned into amphibians and then into dry-land animals; the latter, and maybe also the aquatic animals (e.g. flying fish), gave rise to aerial forms, i.e., flying creatures such as birds, insects and bats. Thus, after the conquest of the air may well come the conquest of the ether: airborne beings may turn into spaceborne beings.

These beings will become the naturalized citizens of the realm of the ether, of pure sunlight and the vast reaches of space.

Reaction-driven devices will make available to mankind limitless living space and will bring within reach two million million times more solar energy than is received on earth.

Furthermore, it would be possible to reach other suns as well, although reaction-propelled vehicles would travel some tens of thousands of years to reach them.

In terms of a human lifespan this is obviously an enormous period of time, but relative to the whole of mankind, or to the lifetime of our sun, it is infinitesimal.

During the long voyage to another star the space travelers will be living in a man-made environment, using a reserve of potential energy accumulated from the sun.

The bulk of mankind will probably never perish, but will just keep moving from sun to sun as each becomes extinguished. After many eons we might possibly come to live on an incipient sun, still in a state of nebular matter and destined to burst into glory at some future date.

\(^7\) [One vent equals 3,500 feet]
If we can ever now glimpse the infinite potentialities of man, then who can
tell what we might expect in some thousands of years, with deeper under­
standing and knowledge.

There is thus no end to the life, evolution and improvement of mankind.
Man will progress forever. And if this be so, he must surely achieve immor­
tality.

So push on confidently forward, workers of the earth, and remember that
every ounce of your efforts is eventually bound to bring a priceless reward.

ROCKETS. GENERAL DESCRIPTION AND STRUCTURE

In various of his works Tsiolkovskii gave descriptions, sometimes together
with drawings, of rockets of increasing complexity.

These types may be classified in ten categories.

1. **Initial rocket, with a straight nozzle.** Described in "Nauchnoe
Obozrenie" (Scientific Review) 1903, No. 5 (without drawing*). The drawing
Corresponding to the description is given in "Vestnik Vozdushnykh
Plavaniy" (Aeronautical Journal) 1911, No. 19, page 17. The description is reproduced
in the journal "Vozdushnykh Plavaniy" (The Aeronaut) 1910, page 114, and also
in the booklet "The Rocket in Space," Kaluga, 1924. The drawing given
in this booklet, however, fits the rocket of the next type mentioned below.

2. **Second rocket, with a bent nozzle.** First published in the booklet "The
Exploration of Space Rocket-Propelled Devices," Kaluga, 1914, which also
contains a drawing. The same drawing later appears in the above-mentioned
booklet of 1924.

3. **Third rocket, with a double hull and pumps.** Published in Ya. Perel'man's book "Interplanetary Voyages," Petrograd, 1915, page 100, and also in
K. Veigelin's paper in the journal "Priroda i lyudi" (Nature and Man) 1914,
No. 4.

4. **Experimental rocket of the year 2017.** Described in the book "Beyond

5. **Compound passenger rocket** of the year 2017. Described in the above,
pages 17, 21, etc.

6. **Portable rocket** harness for personal use. Described in the above,
page 43.

7. **Lunar rocket,** for use on the moon. Described in the above, pages 78–
88.

8. **The rocket described by Scherschevsky.**
9. Cosmic rocket (1926 design).
10. Cosmic rocket trains.

Tsiolkovskii himself provided drawings only for the rockets of the first
and second type, and a sketch of the third rocket type for Perel'man's book.
In the following we give diagrams of the other rocket types as well, drawn
as closely as possible to the author's descriptions.

We now proceed to the description of the rockets.

* It is apparent from the description that a drawing should have been included on page 49; it must have been
omitted in the proofreading.
First type, 1903. Rocket with a straight nozzle

The rocket (Figure 17) has the form of an elongated metal casing, streamlined to reduce drag, supplied with illumination, oxygen, carbon dioxide absorbers and other purifiers; it is designed to carry all kinds of physical instruments, together with a navigator. The casing contains a large reserve of substances, which form an explosive mixture on contact. These substances are steadily fed at a given point where they ignite and form hot gases which flow along flaring tubes, shaped like horns. The tubes lie longitudinally against the wall of the casing. The explosive mixture is produced at the narrow end of the tube, giving rise to strongly compressed burning gases. These gases expand towards the wide end, cooling down in the process, and are expelled through the cone opening with a tremendous exhaust velocity. The fuel reserve is burnt up in 20 minutes.

Figure 17 shows the compartments of liquid gases, hydrogen (H) and oxygen (O). Ignition occurs at the point marked A. The combustion gases escape through the cone opening at B. The nozzle AB is surrounded by a cooling sheath containing rapidly circulating liquid (hydrogen and oxygen) at a temperature of 200–250°C.

To prevent spinning of the rocket during flight, the reaction force must pass through the center of inertia. If the balance is accidentally upset it may be restored either by transferring some mass within the rocket, or else by turning the nozzle cone or a baffle plate in the exhaust stream. In case the stability proves difficult to control manually, use can be made of one of the following automatic steering devices:

1) a magnetic needle*;
2) the intensity of sunlight;
3) gyroscopes.

As the rocket becomes unbalanced, the sun's image obtained by a biconvex lens shifts position within the rocket and causes a gas to expand and close an electrical circuit which brings into motion masses (of which there must be two) restoring the equilibrium.

A gyroscope may be made up of two wheels spinning in different planes, and can serve for directional control by acting on small springs whose deformation closes an electrical circuit causing the displacement of balancing masses.

* Out in space a magnetic needle would of course be useless.
The walls of a nozzle tube made of steel should not be more than 5 mm thick; since this is liable to melt, however (melting point 1,300°C), it is better to use a higher-melting material, for instance tungsten whose melting point is 3,200°C.

Second type, 1914. Rocket with bent nozzle

Figure 18 represents the rocket described by Tsiolkovskii in 1914 ("The Exploration of Space with Rocket-Propelled Devices," Kaluga, 1914).

The left-hand, aft section of the rocket is divided into two compartments by a partition not shown in the drawing. One compartment holds freely evaporating liquid oxygen. The low-temperature oxygen surrounds part of the combustion tube and other components which are subjected to high heat. The other compartment contains liquid hydrocarbons. The two dots near the bottom indicate the cross section of the fuel lines entering the combustion tube. Two ducts issue from the sides of the tube inlet (coiled around the two dots), carrying a high-speed gas flow which sucks liquid fuel and injects it into the tube opening, in the way of a carburetor.

The cool oxygen evaporating from the tank circulates in the free space between the two hulls, thus preventing the rocket interior from overheating due to air friction in the atmosphere.

![Figure 18. The 1914 rocket with a bent nozzle](image)

The combustion tube makes a few turns along the rocket axis and a few more perpendicular to it. This is intended to stabilize the rocket or improve its maneuverability. The rings of swiftly flowing gas take the place of heavy spinning disks.

The right-hand, nose section is sealed off from the rest, and is provided with:

1. Gases and vapor required for breathing.
2. Devices to protect living beings against a fivefold or tenfold increase in the force of gravity. For instance, a passenger may be placed lying in a water-filled container.
3. Food provisions.
4. Controls capable of being operated from a lying position.
5. Absorbents of carbon dioxide and of all other respiratory by-products.

The inside of the tube may be lined with some kind of refractory material, such as carbon, tungsten, or a ceramic. At the low temperature of inter-
planetary space, or at temperatures induced by the evaporation of liquid hydrogen and oxygen, some metals become tougher.

Tsiolkovskii also tentatively calculated the dimensions of the combustion tube: wall thickness (steel) about 4.5 cm, inner diameter 4 cm and outer diameter 13 cm.

Propellant is fed into the tube by the process of combustion itself. For directional stabilization use is made of the arrangement described above. Alternatively, deflection plates may be installed in the escaping gas stream; tilting the plates about the appropriate axis produces a resultant torque counteracting the spin of the rocket.

Subsequently, in his paper "The Space Rocket. Experimental Development," 1927, page 22, Tsiolkovskii gave up the idea of a rocket with a bent nozzle.

Third rocket type, 1915

Ya. Perel'man's booklet "Interplanetary Voyages" (Petrograd, Soikin Press, page 100) gave a description and drawing of the rocket, as furnished by Tsiolkovskii (Figure 19).

![Figure 19. The 1915 rocket](image)

The construction is as follows: the tube A and chamber B are made of a tough high-melting metal and coated inside with a more refractory material, such as tungsten. C and D are pumps feeding liquid hydrogen and oxygen into the combustion chamber. The rocket also has an external refractory hull. Cold gaseous oxygen evaporating from the tank is let through the free space between the inner and outer hull, to prevent overheating by air friction in flight through the atmosphere. The compartments of liquid oxygen and hydrogen are separated by a bulkhead (not shown in the drawing). The pipe J leads the evaporating oxygen into the space between the two hulls; the oxygen seeps out through the openings K. A steering attachment (not shown in the drawing), consisting of two baffle plates at right angles to each other, is mounted at the tube outlet. The exhaust flow can be deflected by means of the plates, thus providing a lateral thrust in the desired direction.*

* Perel'man gives the following specifications for the rocket ("Vestnik Znaniya" (Science Journal), 1928, page 591): tube length 10 m, throat diameter 8 cm, weight 30 kg; power of fuel pump motors 100 hp; temperature at tube inlet 300°C; cone angle 30°.
Fourth type. Experimental rocket of 2017

In his book "Beyond the Earth" (Kaluga, 1920, page 13) Tsiolkovskii gives an account of the construction and flight of spaceships one hundred years in the future.

An experimental rocket should first be built. It has the form of a streamlined hollow metal shell, 20 meters long and 2 meters wide (Figure 20). The interior is adequately lit by an array of small windows. Three narrow tubes run along the walls and emerge at the tail of the rocket. The inside holds machinery in a metal housing and compartments for liquid propellant. The combustion products are discharged through the tubes. A number of levers with dials serve as controls. The rocket accommodates three pilots in padded seats.

![Figure 20. Experimental rocket of 2017](image)

During the first test the rocket stood suspended in the air for 20 minutes, consuming one-hundredth part of its fuel. In the second test it climbed to a height of 5 kilometers.

Fifth type. Compound passenger rocket of 2017

The compound passenger rocket (Figure 21) (K. Tsiolkovskii — "Beyond the Earth," page 19) was made up of twenty simple rockets, each containing a fuel reserve, combustion chamber with an automatic injector, exhaust tube, etc. There was an extra section in the middle, not fitted with a propulsion unit, which served as a crew cabin; it was 20 meters long and 4 meters in diameter. The whole rocket measured 100 meters in length and 4 meters in diameter, and was shaped like a huge spindle.

The purpose of the injectors was to feed a steady supply of propellant into the combustion tube. Their design was similar to the Giffard steam-jet injector. The compound propulsion system made it possible to achieve a relatively light weight coupled with a high, effective thrust. The combustion tubes were coiled in spirals, and flared out towards the inlet*. Some of the spirals ran transversely to the rocket axis, and others parallel to it. The propulsion gases were thus made to spin in two mutually perpendicular planes, and this stabilized the rocket and prevented it from pitching and yawing. The exit cones of the tubes were all aligned in the same direction and were arranged in a helical line around the rocket periphery. The combustion chambers and exhaust tubes were made out of very tough high-melting materials, such as tungsten, and so were also the injectors.

* (This is evidently a mistake — it should be towards the exit.)
FIGURE 21. Compound passenger rocket of 2017
propulsion assembly was enclosed in a chamber cooled by the evaporating liquid of one of the propellant components. The rest of the fuel was contained in other insulated compartments. The hull of the rocket was made up of three envelopes. The inner envelope was of tough metal, with quartz windows covered with ordinary glass, and hermetically sealed doors. The second one was of a refractory material, conducting very little heat. The third, outer envelope was a fairly thin shell of infusible metal. When the rocket traveled at a high speed through the atmosphere the outer shell became incandescent, but the heat was radiated outward and only a small amount penetrated the internal envelopes. Heat was further removed by cold gas continuously passing through a porous layer of low heat conductivity, placed between the two outside shells. The engine thrust could be varied, or stopped and started, by means of multiple injectors. These and other devices were used to steer the craft.

The temperature inside the rocket could be regulated at will by means of valves letting cold gas through the intermediate hull. Special tanks supplied oxygen for breathing. Other devices were used to absorb the wastes released by respiration and perspiration. All these could be regulated as needed. Storage chambers were provided with food and water. There were special airtight suits which could be worn in the vacuum of space or in the unbreathable atmosphere of alien planets. There were also all kinds of instruments and devices designed for specific purposes. Liquid-filled compartments were provided to protect the crew from increased gravity during acceleration. When immersed, the men breathed through a tube extending into the air space of the rocket. The liquid effectively counter-balanced their excess weight during the periods of powered drive. The men could freely move their limbs and felt even lighter than they did on earth; they were like swimmers, or like the olive oil floating in wine in Plato's experiment. This ease of movement enabled them to handle efficiently the rocket controls and to regulate the temperature, the engine thrust, the direction of motion, etc. All these operations could be performed by means of a set of levers within reach in the liquid. There was in addition a special automatic control device on which could be recorded all the commands required for several minutes; during that time the levers did not have to be touched, and the controls carried out by themselves what they had been "ordered" to do. The rocket also carried a supply of wheat grain and fruit and vegetable seeds, to be cultivated in special hothouses launched into space. The building materials for the hothouses were laid in stock beforehand.

The rocket had a capacity of about 800 cubic meters. It could hold 800 tons of water. Less than one-third the capacity (240 tons) was taken up by two fuel liquids. This mass was sufficient to accelerate the rocket 50 times in succession to a velocity enabling it to escape from the solar system altogether, and to decelerate it from that speed the same number of times.

The hull, or rather the body of the rocket with all the equipment, weighed 40 tons. The supplies, instrumentation and hothouses amounted to 30 tons. The crew and other gear weighed less than 10 tons. Thus the rocket together with its contents weighed one-third as much as the propellant. The space intended for the crew, filled with low-pressure oxygen, measured about 400 cubic meters. The rocket was designed to take 20 passengers. This gave each person 20 cubic meters of room, which was quite comfortable with
the atmosphere being constantly purified. The 21 sections communicated by narrow passages. The average size of each section was about 32 cubic meters, but half of this space was taken up by pieces of equipment and fuel, leaving about 16 cubic meters free. The middle sections were larger and each could be used as individual private quarters. On the side walls were mounted windows of clear glass, fitted with shutters on the inside and outside.

When the rocket leaves the atmosphere the external air pressure acts on it no longer, and its walls then have to withstand the outward pressure of the breathing oxygen within. The oxygen pressure was only one-tenth of an atmosphere. The windows had double panes with metal mesh fused into the glass.

In order to provide the passengers with the sensation of weight, a pseudo-gravity could be produced in the rocket by making it rotate about a transverse axis using the propulsion tubes C located at each end.

The rocket was heated by sunlight which was admitted through windows covering an area of 320 square meters, and also by adjusting the area of the dark, heat-absorbing coating on the hull exposed to the sun. The heating surface could be further increased by reflecting the sun's rays onto the shadow side of the rocket with parabolic or plane mirrors. Spherical mirrors of this kind can be used to obtain even higher temperatures than required for heating. It is possible to obtain a temperature as high as 5–6 thousand degrees Centigrade, which is suitable for metallurgical applications.

For cooling the rocket use was made of silvered shutters reflecting the sunlight.

The hothouse. The rocket started out from the earth in the form described above. But when it went into orbit around the earth in space, it was necessary to ensure a steady food supply for the passengers. For this purpose the rocket carried a stock of components prepared for the construction of a long hothouse, in which a variety of edible plants were to be grown in the sunlight. The components were taken out of the rocket through airlocks and assembled next to it by the crew clad in spacesuits. The finished hothouse was placed parallel to the rocket and connected to it by passages with airlocks.

The hothouse measured 500 meters in length and 2 meters in diameter. Throughout its length ran a window about 2 meters wide. The window glass was reinforced with wire mesh embedded in it. The rest of the hothouse shell was made of metal, and the whole assembly weighed 20 tons.

Besides the passages, the hothouse was also connected to the rocket by two slim tubes (a and b in Figure 20); one served to discharge into the hothouse the carbon dioxide and other metabolic wastes collecting in the rocket, and the other delivered into the rocket fresh oxygen and nitrogen released by the plants. Inside the hothouse was installed a metal container, running along the whole length. It was filled with semiliquid soil and was pierced with many holes through which seeds and sprouts were planted. The container was internally irrigated. Running along its length at the interior were two pipes, also with a series of holes in them, one of which supplied the soil with gases and the other with fertilizing solution. Both the gases and the liquid fertilizer were fed into the pipes by pumps.
The vapor and gas pressure in the hothouse was so low (20 mm of mercury) that the walls could be made as thin as ordinary plate glass. The soil container could be rotated for uniform illumination. Both the rocket and the hothouse automatically assumed in flight the optimal orientation relative to the sun's rays. This was accomplished through control devices which reacted to the intensity of sunlight, as well as by some other means described further on. The crew entered the hothouse in spacesuits. The water-vapor pressure in it was not more than 4-10 mm, as the evaporation from the soil and foliage was condensed, before reaching saturation, in special attachments that were always kept in the shade and thus stayed at a near-zero temperature. The carbon dioxide, oxygen, nitrogen and other gases were also at a very low pressure.

Self-contained habitable hothouse (Figure 22). When people had become accustomed to living in space on board rockets with hothouses attached to them orbiting the earth, it was decided to build larger hothouses which also circled the earth like satellites, suitable not only for growing plants but also for habitation. The building materials were flown on rockets from earth, and workmen in spacesuits welded the parts of the hothouse together in its future orbit.

The hothouse had a length of 1,000 meters and a width of 10 meters. It was designed for 100 inhabitants. The side facing the sun was transparent over one-third of its circumference. The back part was of metal, with small widely spaced viewports. The transparent front had an extremely strong silvery wire mesh embedded in it, which enabled it to withstand safely the pressure of the breathing atmosphere as well as severe impacts; the other, metal-made part was even stronger. The internal temperature could be controlled both on the inside and outside, and could be varied from 200° below to 100° above zero Centigrade. This was essentially done by altering the emissive power of the exterior of the cylindrical shell. The opaque part of the hothouse was black, but it was provided with another folding envelope which was highly reflecting on either side. When this was stretched over the black surface it cut down the radiation of heat from two-thirds of the cylinder surface, while the incoming sunlight kept heating the hothouse, raising its temperature to 100°C. The opposite occurred when the shiny envelope was folded away; the black metal surface was then exposed and radiated freely into space, causing the hothouse temperature to drop. The temperature could be lowered even more, a reflecting cover being employed to shield the transparent front from the sun's radiation; it could drop as low as 200°C below zero. It could be further reduced or raised by bringing into operation a third, internal surface. As with the previously described hothouse, along the central axis were laid soil tanks and pipes carrying gases and fertilizer for
47 plant growing. The hothouse cylinder was divided lengthwise along a diametric plane by a silvery mesh. The sunlit half was partly shaded by grape vines and other fruit plants winding across the glass front. It was intended for all the dwellers, irrespective of age or sex. The other half was shaded by a thick layer of lush vegetation. It had only some widely spaced windows through which could be seen the black starry sky, the moon, and the earth, which was a thousand times brighter than the moon. This was the all-metal part of the hothouse, in which were laid out 200 separate compartments. One hundred of these were for the use of families, 50 were for single men and 50 for single women. Between the compartments and the main assembly hall there were six long communal rooms, for married people, for boys and for girls. The main hall measured 1,000 × 10 × 5 meters. The size of the private living quarters was 2⅛ × 9 × 5 meters, and the floor ran over the central tank. The communal rooms were 2⅛ × 167 × 10 meters. It should be noted that the dimensions of length, width and height were meaningless in the absence of gravity and became relevant only if the hothouse rotated about a given axis. At the ends of the hothouse were located lavatories and bathrooms.

The humidity in the hothouse was regulated by means of a specially built cooler, consisting of a black metal pipe running the length of the hothouse in the shade. By cycling the outside air through it, the moisture would cool down and precipitate in it, and in this way excessive humidity was avoided. The condensed water was collected in the lavatories and bathrooms, where it was duly purified and used for washing. It was then drained into the soil tank again.

Ventilation was provided by special fans.

Besides long cylindrical hothouses, aggregates of them were also built, e.g. in star-shaped formation (Figure 22 right). Later on these hothouses were joined together in whole clusters and colonies, interconnected by passages with airlocks.

Sixth type. Portable rocket harness

In order to enable spacesuited men to move about outside the rocket, Tsiolkovskii proposed equipping them with compact rocket tubes worn on harnesses, which could be fired at will in any direction. A meter indicates the fuel consumption.

Seventh type. Lunar rocket

For transportation on the moon Tsiolkovskii proposed using a small rocket for two people (Figure 23). The rocket should be mounted on wheels and driven by stored energy, since on the moon the sun's energy would not always be available. The wheels would enable the rocket to travel over flat ground; for crossing peaks and crevasses, auxiliary propulsion tubes are provided, which would lift the rocket against the weak lunar gravity. There is an electric stove inside for heating. The exit hatch is fitted with an airlock. The rocket is ellipsoidal in form (with an elongation ratio of 3)
rather than drop-shaped; since it would be launched from an orbiting station out in space and not from the earth, it would not have to fly through the air.

![Lunar rocket](image)

**FIGURE 23. Lunar rocket**

For trips on the moon's surface, a platform could be installed atop the rocket, on which the travelers might be seated in spacesuits and enjoy the view.

**Eighth type. The rockets described by Scherschevsky**

In his paper "The Space Ship" (Flugsport, 1927, page 425) Scherschevsky gives a cross-sectional drawing of a rocket ship, made according to Tsiolkovskii's 1903 and 1915 descriptions. The drawing is reproduced here (Figure 24). In another work of his, "Die Rakete" (1929, page 123), Scherschevsky gives the following specifications for Tsiolkovskii's rockets (Figure 25). Length-to-diameter ratio, about 10; the inner hull is separated from the outer one by a vacuum (similarly to a Thermos flask); $f$ - fuel, $c$ - hull, $h$ - steering gear, $a$ - hull, $i$ - evacuated space, $d$ - cabin, $i$ - pumps, $e$ - periscopes.

![Rocket diagram](image)

**FIGURE 24. The 1927 rocket**

![Rocket diagram](image)

**FIGURE 25. The 1929 rocket**
For takeoff use is made of a launching rocket (Figure 26) sliding over a trough-shaped track, \( g \), on a cushion of compressed air; \( a \) — space rocket, \( e \) — launching rocket, \( f \) — thrust tubes, \( d \) — fuel, \( c \) — periscope.

![FIGURE 26. Launching rocket](image)

**Experimental preparation for space flights**

In September 1927 Tsiolkovskii published a new booklet, "The Space Rocket. Experimental Development," in which, as the title indicates, he gives a plan of laboratory tests that have to be run before space flight. The contents of the book are as follows.

1. **Layout of a stationary rocket power plant.**

   Figure 27 is a diagrammatic drawing (not to scale) of a rocket testing laboratory.

![FIGURE 27. Rocket testing laboratory](image)

On the right is located a motor for pumping liquid air, oxygen or its endogenous compounds, and hydrogen compounds. HP and OP are pumps feeding oxygen and hydrogen compounds into the propulsion tube, of a
capacity varying with the kind of propellant employed; K – pump valves; OFP and HFP – pipes supplying fuel from the tanks to the pumps; OG and HG – grilles with slanting perforations for thorough fuel mixing; PT – cone shaped propulsion tube, made up of two shells; an inner tough refractory lining, and an outer reinforcing heat-conducting sheath; OT and HT – oxygen and hydrogen (or kerosene) tanks. The baffle plates face the exhaust opening. All the components are fastened to the frame and support which receive the reaction thrust and transmit it to the spacecraft. The propulsion tube is made of steel.

**FIGURE 28. Rocket testing laboratory**

2. **Dimensions and quantities.** Assuming a craft of one ton total weight, Tsiolkovskii estimates that flight should be possible with a fuel consumption of 0.3 kg per second. He actually carries out his calculations for one kg per second. The propulsion tube inlet diameter is taken as 2–4 cm, the instantaneous pressure thrust (during explosion) 12–48 tons, with a mean of about one ton. The propulsion tube is calculated for a thrust of 3 tons, at a weight of about 151 kg. The weight of the motor with the pumps and piping is about 10 kg, and the weights of the frame, tanks, steering gear and pilot amount to another 140 kg. The fuel weighs 700 kg.

3. **Fuel.** The propellant mixture consists of an oxidizing agent and a hydrogenous compound. The first may be liquid air, or nitric anhydride (N₂O₅), or liquid oxygen. For the second, use may be made of benzene or kerosene, or alternatively of a metal (in liquid form) or liquid hydrogen (ideally).

4. **Operation of the motor during testing.** This part deals with the way in which the rocket drive is supposed to function, and the safety measures that have to be taken when running tests.

In conclusion Tsiolkovskii raises some objections to Lademann's paper appearing in the German journal "ZFM" (28 April, 1927), which gives an analysis of the rocket design.
Note. In his paper ("ZFM" 1927, page 482) Lademann reviews in German the above project and includes a more precise drawing of the laboratory plan (Figure 28).

Ninth type. Cosmic rocket (1926 design)

Tsiolkovskii did not provide any drawings for his cosmic rocket, but he gave a fairly detailed description of it, on the basis of which some schematic drawings have been made here (Figure 29). The basic design is as follows: the rocket shell is a streamlined body of revolution, with an elongation ratio [fineness ratio] of 10:1. At the stern are mounted three control surfaces for steering the rocket either in the atmosphere or out in space; one acts as rudder (1), one as elevator (2), and one is for roll control (3). The rocket is additionally stabilized by ailerons in its sides (4).

![Figure 29. The 1926 cosmic rocket](image)

![Figure 30. The 1926 compound rocket](image)

The control surfaces operate under air pressure during flight in the atmosphere, and by the action of the exhaust gas stream when the rocket is moving in space. Flap 1 turns about a vertical axis, flap 2 turns about a...
horizontal transverse axis, and flap 3 turns about the longitudinal axis of the rocket. The propulsion tube is cone shaped, with an apical angle of 1–5°. The cabins are fitted with quartz windows coated on the inside with a substance screening out the harmful solar radiation. There is a valve set in the rocket hull, which automatically opens if the internal pressure rises above the level required for breathing (partial oxygen pressure 0.2 atmosphere).

Figure 30 shows a compound rocket made up of several single rockets joined side by side, with reinforcing fins along the joints. This shape allows the rocket to glide more easily through the atmosphere.

The following table gives the hull weight of single rockets made of a strong iron alloy, given a safety factor of 4 for an internal-external pressure differential of one atmosphere.

<table>
<thead>
<tr>
<th>Rocket capacity ( \text{m}^3 )</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Weight of internal gas at air density ( \text{kg} )</th>
<th>6.5</th>
<th>13</th>
<th>19.5</th>
<th>26</th>
<th>39</th>
<th>62</th>
<th>65</th>
<th>130</th>
</tr>
</thead>
</table>

| Hull weight \( \text{kg} \) | 33 | 65 | 98 | 130 | 195 | 260 | 325 | 650 |

The maximum load for any rocket of given capacity is approximately expressed in tons by the first line in the table. For small rockets the hull will be heavier than listed above. The most suitable fuel is a liquid hydrocarbon with a high hydrogen content. The oxygen for combustion should be in the form of endogenic compounds, which release large amounts of heat on dissociation. The liquid propellants have to be kept in containers with openings through which the forming gases can freely escape. The gases delivered to the combustion chamber must be under a pressure of about 100 atmospheres, thus not requiring much pumping work. The table below gives the performance data of a reaction drive, assuming a propellant density of one and a rocket mass of one ton.

<table>
<thead>
<tr>
<th>Rocket velocity, ( \text{km/sec} )</th>
<th>8</th>
<th>11</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propellant mass, tons</td>
<td>4</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Combustion time, sec ( (p=10) )</td>
<td>800</td>
<td>1,100</td>
<td>1,700</td>
</tr>
<tr>
<td>Fuel consumption, ( \text{kg/sec} )</td>
<td>5</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>Pumping work, ( \text{kg-m} )</td>
<td>600</td>
<td>1,100</td>
<td>1,700</td>
</tr>
<tr>
<td>Combustion time, sec ( (p=1) )</td>
<td>8,000</td>
<td>11,000</td>
<td>17,000</td>
</tr>
<tr>
<td>Fuel consumption, ( \text{kg/sec} )</td>
<td>0.5</td>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Pumping work, ( \text{kg-m} )</td>
<td>50</td>
<td>110</td>
<td>170</td>
</tr>
</tbody>
</table>

The ratio of the cross-sectional areas of the propulsion tube near the combustion chamber \( \omega_2 \) and at the outlet \( \omega_1 \) is given by

\[
\frac{\omega_1}{\omega_2} = \left(1 + \frac{l}{r_1 \tan \alpha}\right)^2;
\]

where \( l \) is the tube length, \( r_1 \) is the radius of \( \omega_2 \), and \( \alpha \) is the cone angle.
Listed below are the results obtained from this relationship:

<table>
<thead>
<tr>
<th>Angle, degrees</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{m_1}{m_2} )</td>
<td>28.8</td>
<td>95.1</td>
<td>199</td>
<td>342</td>
<td>524</td>
<td>740</td>
<td>1,296</td>
<td>2,000</td>
</tr>
<tr>
<td>Diameter ratio</td>
<td>5.37</td>
<td>9.75</td>
<td>14.1</td>
<td>18.5</td>
<td>22.9</td>
<td>27.2</td>
<td>36.0</td>
<td>44.7</td>
</tr>
<tr>
<td>Outlet diameter, m</td>
<td>0.22</td>
<td>0.39</td>
<td>0.56</td>
<td>0.74</td>
<td>0.92</td>
<td>1.08</td>
<td>1.44</td>
<td>1.8</td>
</tr>
</tbody>
</table>

The weight of the motor pumping propellant into the combustion chamber (in intermittent operation) varies between 5 and 100 kg. The rocket orientation during powered flight may be indicated by a set of spinning disks.

Projected organization of space flights

1. Tests with a reaction-driven aeroplane [jet-propelled airplane], gradually increasing the speed and altitude and reducing the wing area.
3. Passing over to rockets.
4. Rocket flight with a velocity of up to 8 km/sec outside the atmosphere. Rocket built along the lines of Figure 26.
5. Flights farther out. Developing closed food-cycling and waste disposal systems, and penetrating deep into space.
6. Setting up space colonies in the asteroid belt and spreading out into the Galaxy.

Tsiolkovskii concludes with an account of what a man might experience when he first lands on an asteroid 6 km in diameter, where the gravity is 2,250 times lower than on earth, then on an asteroid 56 km in diameter, and finally on an asteroid of 560 km diameter, where the gravity is 22.5 times less than on earth.

Cosmic rocket trains

In 1929 Tsiolkovskii put out a new book entitled "The Space-Rocket Train." The rocket train is a compound rocket, or rather an assembly of rockets, which starts on a ground run, then lifts off and finally goes into space. A five-stage rocket train, for instance, is first propelled by the leading rocket until its fuel is used up; the first rocket is then jettisoned, and the second one fires in turn. This is successively repeated down to the fifth stage which attains a velocity high enough to escape into space. The reason for having the driving rocket in the lead is that the assembly tends to resist the resultant tensile stresses better than compressive ones.

Each rocket has a length of 30 meters and a diameter of 3 meters. The exhaust flow emerges at an angle to the rocket axis, in order not to impinge
on the rear of the rocket. The ground run extends over some hundreds of kilometers.

Figure 31 shows the side and top view of a train consisting of five triple rockets. Wall thickness, 2 mm; hull weight, 4 1/2 tons; contents, 4 1/2 tons; fuel, 27 tons. The rocket weight thus totals 36 tons. There are four or more stern tubes, set at an angle so as to keep the propulsion gases clear of the rear of the rocket. The windows are made of quartz.

In his book Tsiolkovskii calculates the fuel and rocket mass and the run and takeoff velocities for rocket trains with a different number of component stages. Further he computes the propulsion time, the paths, and takeoff angles.

**REACTION-DRIVEN AEROPLANE**

In 1930 Tsiolkovskii published a little booklet, "The Reaction-Driven Airplane," in which he describes the design of an aircraft propelled by the reaction of gases. According to his calculations, such an aeroplane flying at an altitude of about 37 kilometers would be able to develop a speed of 3,600 km/hour and stay in the air for about an hour.

For propulsion he proposed using a conventional aircraft engine, with the cylinders replaced by conical tubes through which the combustion products are expelled backwards. The air for combustion is sucked in from outside and compressed, thus heating up considerably. In order to cool it down, it is passed through jackets around the exhaust openings at the rear, which are strongly cooled by the expansion of the escaping gases. The cool compressed air then enters the combustion chamber of the engine. The fuel may be gasoline.
Chapter IV

THEORY OF ROCKET FLIGHT

This chapter summarizes the theory of flight, ascent, and descent of a rocket as it was developed by K. Tsiolkovskii in the following publications:

1. "Exploration of Space with Rocket-Propelled Devices (Issledovanie mir-ovykh prostranstv reaktivnymi priborami). Published in the journal "Nauchnoe Obozrenie" (Scientific Review), No. 5, p. 44, 1903.

2. "A Rocket-Propelled Device as a Means of Flight in Vacuum and in the Atmosphere" (Reaktivnyi pribor kak sredstvo poleta v pustote i v atmosfere). Published in the journal "Vozdushnoplavatel'" (The Aeronaut) p. 110, 1910.

3. "Exploration of Space with Rocket-Propelled Devices" (Issledovanie mir-ovykh prostranstv reaktivnymi priborami). (The rocket-propelled device is K. Tsiolkovskii's "rocket.") Published in the journal "Vestnik vozdukhoplavaniya" (Aeronautical Journal), Nos. 19, 20, 21, 22, 1911.

4. Same article continued in the same journal, Nos. 2, 3, 5, 6-7, 9, 1912.


7. "The Rocket in Space" (Raketa v kosmicheskom prostranstve). Kaluga, 1924. (Identical with No. 1, except for minor additions)


We see from the above list that K. Tsiolkovskii first published his theory of rocket flight in 1903.

To avoid unnecessary and lengthy repetitions, we will refer to the different publications in what follows by citing a fractional number, in which the numerator identifies the particular reference from the above list (from 1 to 8) and the denominator gives the page number.

1. THE PRINCIPLE OF ROCKET FLIGHT (5/8 and 6/34)

Consider two spheres in gravityless free space with a compressed spring between them. If the spring is allowed to stretch, one of the spheres
will be displaced to the right, whereas the other will move to the left. The work done by the spring in displacing the spheres will be divided in equal shares between the two spheres. A similar situation is observed when two rubber balls are pressed one against the other and then released. They will recoil without any spring between them. Now consider a tube with compressed gas. If the tube is opened at one of its ends only, the gas will exert pressure on the other end which remains closed and the tube will be propelled by this pressure to the right, say. The gas will then escape from the tube to the left. A similar situation is observed when a rifle or a gun is fired.

The material environment or the atmosphere in which the instrument is immersed is a factor of secondary importance, although it may affect the magnitude and the intensity of the reaction forces developing in the system.

In what follows, we will describe the theory of rocket motion as it was published by Tsiolkovskii in 1903, using references Nos. 1 through 8.

2. THE FLIGHT OF A ROCKET IN A MEDIUM WITHOUT GRAVITY AND AIR (1/52, 7/9, 3/19)

Notation:
\( M_1 \) - mass of projectile, minus the propellant (explosives);
\( M_2 \) - total mass of propellant;
\( M \) - varying instantaneous mass of unexploded propellant remaining in the projectile at a given instant;
\( V_1 \) - velocity of exploding elements relative to the rocket;
\( dM \) - an infinitesimal mass of propellant ejected from the rocket nozzle with constant velocity \( V_1 \).
\( V \) - the variable velocity of the rocket;
\( dV \) - rocket velocity increment.

The initial total mass of the rocket is \( M_1 + M_2 \).
While the propellant is being exploded, the rocket mass is variable, \( M_1 + M \).
When all the propellant has exploded, the final rocket mass is fixed, \( M_1 \).

To impart the maximum velocity to the rocket, the products of explosion should be ejected in a fixed direction relative to the stars. To achieve this, the rocket must not turn, so that the resultant of the propelling forces must pass through the center of pressure of the propellant and through the center of inertia of the entire system of moving masses.

Assuming this optimal ejection of the propelling gases in a fixed direction, we obtain the following differential equation, which follows from the law of conservation of momentum:

\[
dV (M_1 + M) = -V_1 dM
\]  (1)

Separating the variables and integrating, we find

\[
\frac{1}{V_1} \int dV = - \int \frac{dM}{M_1 + M} + C,
\]  (2)

whence

\[
\frac{V}{V_1} = - \ln \left( \frac{M_1 + M}{M_1} \right) + C.
\]  (3)
Here \( C \) is a constant which is determined from the initial conditions, i.e., \( M = M_1 \) and \( V = 0 \);

\[
C = \ln \left( \frac{M_1 + M_2}{M_1 + M} \right)
\]  \hspace{1cm} (4)

Equation (3) thus takes the form

\[
\frac{V}{V_i} = \ln \left( \frac{M_1 + M_2}{M_1 + M} \right)
\]  \hspace{1cm} (5)

The maximum velocity is attained by the projectile when the entire propellant has exploded, i.e., when \( M = 0 \). Then

\[
\frac{V}{V_i} = \ln \left( 1 + \frac{M_2}{M_1} \right)
\]  \hspace{1cm} (6)

Equation (6) shows that the projectile velocity \( V \) increases indefinitely as the quantity of the propellant \( M_2 \) increases, although the rate of increase progressively diminishes.

Tsiolkovskii suggested using as propellant a mixture of hydrogen and oxygen. These elements combining in the gaseous state to give 1 kg of water release 3,825 calories (1 calorie is the quantity of heat required to raise the temperature of 1 kg of water by 1°C), the corresponding work being \( 3,825 \times 424 = 1,621,800 \) kg·m. Assuming constant gravitational forces, we find that this work is equivalent to the work done by a mass of 1 kg moving with the velocity defined by the equation

\[
\frac{mV^2}{2g} = 1,621,800;
\]

where

\[
m = 1; \quad g = 9.81 \text{m/sec};
\]

hence

\[
V_i = 5,700 \text{m/sec}.
\]

Inserting this result in (6), we can find the projectile velocity as a function of the ratio \( \frac{M_2}{M_1} \). However, increasing the rocket velocity requires a disproportionately large increase of the propellant mass \( M_2 \). Therefore, while it is quite simple to increase the mass of the projectile that we intend to launch into space, we may find it difficult to achieve the required velocities.

The results of calculations using equation (6) are summarized in Table 1. We see from the table that quite substantial velocities can be attained; for example, for \( \frac{M_2}{M_1} = 193 \), \( V = 30,030 \) m/sec which is almost equal to the orbital velocity of the Earth. For \( \frac{M_2}{M_1} = 1 \), \( V \) is almost double the escape velocity from the moon and exceeds the escape velocities from Mars or Mercury. For \( \frac{M_2}{M_1} = 3 \), the velocity is nearly equal to that which is needed to place the projectile in a satellite orbit around the earth. For \( \frac{M_2}{M_1} = 6 \) the
velocity is sufficient to allow the projectile to escape from the earth and to continue orbiting around the sun as an independent planet, etc.

(58) TABLE 1.

<table>
<thead>
<tr>
<th>$\frac{M_2}{M_1}$</th>
<th>$\frac{V}{V_1}$</th>
<th>$V, m/sec$</th>
<th>$\frac{M_2}{M_1}$</th>
<th>$\frac{V}{V_1}$</th>
<th>$V, m/sec$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.095</td>
<td>543</td>
<td>7</td>
<td>2,079</td>
<td>11,800</td>
</tr>
<tr>
<td>0.2</td>
<td>0.182</td>
<td>1,037</td>
<td>8</td>
<td>2,197</td>
<td>12,500</td>
</tr>
<tr>
<td>0.3</td>
<td>0.262</td>
<td>1,493</td>
<td>9</td>
<td>2,303</td>
<td>13,100</td>
</tr>
<tr>
<td>0.4</td>
<td>0.336</td>
<td>1,915</td>
<td>10</td>
<td>2,398</td>
<td>13,650</td>
</tr>
<tr>
<td>0.5</td>
<td>0.405</td>
<td>2,308</td>
<td>19</td>
<td>2,996</td>
<td>17,100</td>
</tr>
<tr>
<td>1</td>
<td>0.683</td>
<td>3,920</td>
<td>20</td>
<td>3,044</td>
<td>17,330</td>
</tr>
<tr>
<td>2</td>
<td>1,098</td>
<td>6,260</td>
<td>30</td>
<td>3,434</td>
<td>19,560</td>
</tr>
<tr>
<td>3</td>
<td>1,386</td>
<td>7,880</td>
<td>50</td>
<td>3,932</td>
<td>22,400</td>
</tr>
<tr>
<td>4</td>
<td>1,699</td>
<td>9,170</td>
<td>100</td>
<td>4,612</td>
<td>26,280</td>
</tr>
<tr>
<td>5</td>
<td>1,792</td>
<td>10,100</td>
<td>193</td>
<td>5,268</td>
<td>30,030</td>
</tr>
<tr>
<td>6</td>
<td>1,946</td>
<td>11,100</td>
<td>Infinite</td>
<td>Infinite</td>
<td>Infinite</td>
</tr>
</tbody>
</table>

For lower values of the ratio $\frac{M_2}{M_1}$, the projectile may nevertheless ascend to high altitudes above the ground. For $\frac{M_2}{M_1} = 0.1$, we have $V = 543$ m/sec and the projectile will rise to around 15 km altitude.

We see from the table that with small fuel reserves the final velocity is roughly proportional to the total fuel mass ($M_2$), so that the altitude of ascent in this case is proportional to the square of the fuel mass ($M_2$).

The ratio of the work done by the rocket to the work of the explosive propellant will be called the utilization of the rocket. Let us now compute the utilization. The work done by the propellant is

$$\frac{V^2 \cdot M_2}{2g},$$

where $g$ is the terrestrial gravitational acceleration.

The work done by the rocket is

$$\frac{V^2 \cdot M_1}{2g},$$

or from equation (6)

$$\frac{V^2 \cdot M_1}{2g} \left[ \ln \left( 1 + \frac{M_2}{M_1} \right) \right]^2.$$

The utilization is therefore equal to

$$\frac{M_1}{M_2} \left[ \ln \left( 1 + \frac{M_2}{M_1} \right) \right]^2. \quad (7)$$

Table 2 lists the results of computations for various values of $\frac{M_2}{M_1}$. We see from the table and from equation (7) that for very small propellant
reserves, the utilization is equal to \( \frac{M_f}{M_i} \), i.e., it decreases the relative quantity of the propellant. This follows from the following expansion:

\[
\ln (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}.
\]

so that approximately

\[
\frac{M_f}{M_i} \left[ \ln \left(1 + \frac{M_2}{M_i} \right) \right] = \frac{M_f}{M_i} \cdot \left( \frac{M_2}{M_i} \right)^2.
\]

<table>
<thead>
<tr>
<th>( \frac{M_2}{M_i} )</th>
<th>Utilization</th>
<th>( \frac{M_f}{M_i} )</th>
<th>Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.090</td>
<td>7</td>
<td>0.62</td>
</tr>
<tr>
<td>0.2</td>
<td>0.165</td>
<td>8</td>
<td>0.60</td>
</tr>
<tr>
<td>0.3</td>
<td>0.223</td>
<td>9</td>
<td>0.59</td>
</tr>
<tr>
<td>0.4</td>
<td>0.282</td>
<td>10</td>
<td>0.58</td>
</tr>
<tr>
<td>0.5</td>
<td>0.328</td>
<td>19</td>
<td>0.47</td>
</tr>
<tr>
<td>1</td>
<td>0.48</td>
<td>20</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>30</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
<td>50</td>
<td>0.31</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>100</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>0.64</td>
<td>193</td>
<td>0.144</td>
</tr>
<tr>
<td>6</td>
<td>0.63</td>
<td>Infinite</td>
<td>Zero</td>
</tr>
</tbody>
</table>

As \( \frac{M_2}{M_i} \) is increased, the utilization increases reaching its maximum for \( \frac{M_2}{M_i} = 0.65 \), and then starts decreasing. On the whole, for \( \frac{M_2}{M_i} = 1-20 \), the utilization is fairly high, being equal to around 0.5.

3. VERTICAL FLIGHT OF A ROCKET UNDER THE INFLUENCE OF GRAVITY

We saw that without gravity a rocket may reach very high velocities. It may achieve the same velocities in a gravitational field, provided the propellant explosion is instantaneous. Instantaneous explosion, however, is impracticable, since the tremendous jolt it is bound to produce will prove unbearable to human beings or material objects inside the projectile. Slow, gradual explosion of the propellant is needed. Gradual explosion, however, reduces the propulsion effect. It may in fact drop to zero when the constant acceleration \( p \) of the rocket becomes equal to the earth's gravitational acceleration \( g \), and the projectile will remain stationary in the air, without any supports.

When a rocket moves in gravityless free space, the time \( t \) for the entire propellant to explode is

\[
t = \frac{V}{p};
\]
here $V$ is the final velocity of the projectile when all the fuel has burnt out.

The quantity of propellant burnt in unit time under these conditions steadily decreases in proportion to the decrease in the total mass of the projectile with the diminishing propellant reserve. Given the free-space acceleration $p$, we may compute the (instantaneous) magnitude of gravitation inside the rocket in the course of its accelerated motion.

Taking the acceleration at the ground to be 1, we find the instantaneous, variable gravitation $\frac{p}{g}$ in the projectile. This ratio gives the increased pressure of the objects in the rocket on their supports. Let $V$ be the final velocity of the rocket in a medium with constant gravitational acceleration $g$. The time for the entire fuel reserve to burn out is then the same as in the free-space case:

$$t = \frac{V}{p - g};$$

here it is assumed that $p$ and $g$ are antiparallel.

If $p = 0$, we have $\frac{p}{g} = 0$. This corresponds to the case of no propellant. The projectile moves by inertia, and all the objects inside the projectile are weightless, i.e., their weight cannot be detected by a spring balance.

If $p = g$, we have $\frac{p}{g} = 1$. This is the case when the rocket is maintained stationary in the air by the reaction of the exploding propellant. The relative gravitation inside the projectile is then equal to the earth's gravitation. If the projectile had some initial velocity (upward, sideward, etc.) prior to the explosion, it will continue its motion by inertia.

We have from (8) and (9)

$$V = V_1 \frac{p}{p - g};$$        \hfill (10)

so that, given the final velocity of the rocket, we can find $V$ and then, using (6), compute the quantity of propellant needed in this particular case.

Equations (6) and (10) yield

$$V_2 = V_1 \left(1 - \frac{g}{p}\right) \ln \left(1 + \frac{Mf}{M_0}\right).$$        \hfill (11)

Equations (10) and (11) show that gravitation reduces the velocity of the rocket, which drops to zero when $\frac{p}{g} = 1$. The higher $\frac{p}{g}$, the higher is the velocity of the rocket. However, $\frac{p}{g}$ cannot be increased without limit, since beyond a certain point the crew and the passengers are in danger.

In the limit, when $\frac{p}{g}$ is infinitely large, equation (11) reduces to equation (6):

$$V_2 = V_1 \ln \left(1 + \frac{Mf}{M_0}\right),$$

i.e., for instantaneous explosion of the entire propellant reserve, the rocket velocity $V_1$ in a gravitational field is equal to the free-space velocity.
The length of burning of the propellant is obtained from equation (9):

1) For $V = 11,100 \text{ m/sec}, \frac{M_2}{M_1} = 6$, we have $t = 1,133 \text{ sec} = 19 \text{ min}$, i.e., in free space the rocket would have moved with uniform acceleration during 19 min and in a gravitational field it would have remained stationary for the same length of time.

2) For $V = 3,920 \text{ m/sec}, \frac{M_2}{M_1} = 1$, we have $t = 400 \text{ sec} = 6^{2/3} \text{ min}$.

3) For $V = 543 \text{ m/sec}, \frac{M_2}{M_1} = 0.1$, we have $t = 55.4 \text{ sec}$.

Thus, if a given quantity of propellant is allowed to burn too gradually, there is a danger than it will burn out without producing any useful effects. This low-intensity burning of the propellant may only prove useful for horizontal flight, with the rocket covering a large distance in a short time. (Flight outside the atmosphere is also conceivable.)

The smaller $\mu$, the longer is the time the rocket may remain in the field of gravitation. Thus, on the moon, for $\frac{M_2}{M_1} = 6$, we have $t = 2 \text{ hrs}$.

If the acceleration is 10 times the normal gravitational acceleration, i.e., $\frac{p}{g} = 10$, and $\frac{M_2}{M_1} = 6$, we have $V_i = 9,990 \text{ m/sec}$ and equation (8) gives $t = 113 \text{ sec}$.

Let us now compare the work done by the propellant in free space with the same work done when the rocket flies in a gravitational field.

The acceleration in a gravitational field is $p_i = p - g$. Suppose that $g$ remains constant to altitudes of a few hundred km. During the burning time $t$ the rocket will then ascend to the altitude

$$h = \frac{1}{2} p_1 \cdot t = \frac{p - g}{2} t$$  \hspace{1cm} (12)

and using (9) we find

$$h = \frac{V_i^2}{2(p - g)}.$$ \hspace{1cm} (13)

Inserting for $V_i$ its expression from (10), we find

$$h = \frac{p - g}{2p} V_i = \frac{V_i^2}{2p} (1 - \frac{g}{p}).$$ \hspace{1cm} (14)

The useful work done by the rocket in free space is

$$T = \frac{V_i^2}{2g}.$$ \hspace{1cm} (15)

The corresponding work in the gravitational field is

$$T_i = h + \frac{V_i^2}{2g}.$$ \hspace{1cm} (16)

The ratio is

$$\frac{T_i}{T} = \frac{2h + V_i^2}{V_i^2}.$$ \hspace{1cm} (17)
Eliminating \( h \) and \( V_2 \), we obtain

\[
\frac{T_i}{T} = 1 - \frac{\rho}{g} \tag{18}
\]

i.e., the work in the gravitational field, produced by a given quantity of propellant \( M_1 \), is less than the free-space work. The difference \( \frac{\rho}{g} \) decreases as the gas ejection velocity increases and as \( \rho \) is made higher.

For the previous case \( \left( \frac{M_1}{M_0} = 6; \frac{\rho}{g} = 10 \right) \) we have a loss of 0.1 and a utilization of 0.9. When \( \rho = g \) and the projectile is stationary in the air, the loss is one and the utilization is 0. The utilization remains zero when the projectile has a constant horizontal velocity.

For \( \frac{M_1}{M_0} = 6 \) and \( \frac{\rho}{g} = 10 \), the ascent altitude is \( h = 565 \) km.

The velocity \( V_2 \) is also less than \( V \), as we see from (10):

\[
V_2 = V \left( 1 - \frac{\rho}{g} \right).
\]

This relation is analogous in its form to (18).

We see from (18) that

\[
T = T_1 \left( \frac{\rho}{g} \right). \tag{19}
\]

If \( T_1 \) is known, \( T \) can be found and we can then determine \( V \) for free-space conditions:

\[
T = M_1 \frac{V^2}{2g}.
\]

Once \( V \) has been found, we can determine the propellant mass \( M_2 \) from (5):

\[
M_2 = M_1 \left( e^{\sqrt{\frac{T_i}{T_1} \frac{\rho}{g} - 1}} \right), \tag{20}
\]

where

\[
T_2 = M_1 \frac{V^2}{2g}.
\]

Here \( g \) is a constant deceleration factor equal to the sum of the gravitational acceleration and the drag of the medium. The gravitational acceleration gradually decreases with altitude, and the utilization correspondingly increases. On the other hand, the atmospheric drag (although, as we shall see, it is very small compared to the weight of the projectile) acts to reduce the utilization.

The latter factor is more than compensated by the utilization increment associated with the decrease in gravitation. Equation (20) therefore may be applied to vertical flight of a rocket.
4. BRAKING OF A ROCKET IN FREE SPACE

Consider a rocket which has been accelerated to a velocity of 10,100 m/sec in free space by burning some of its propellant reserve (Table 1). To bring the rocket to a standstill, it should be imparted the same velocity, but in the opposite direction.

According to Table 1, the remaining quantity of the propellant should be 5 times the mass of the rocket \((M_1)\). Therefore, at the end of the first acceleration phase, the propellant reserve should be \(M_2 - 5M_1\) and the total rocket mass is \(6M_1\). To impart a velocity of 10,100 m/sec to this mass during the initial acceleration stage, we require a quantity of propellant equal to \(6M_1\times5 = 30M_1\), and this together with the remaining reserve of 5 \(M_1\) gives a starting quantity of 35 \(M_1\).

Let \(\frac{M_2}{M_1} = q\) and let \(M_3\) be the total quantity of the propellant needed for the acceleration and the deceleration (in our case, \(M_3 = 35M_1\)).

Then

\[
\frac{M_2}{M_1} = q + (1 + q) \cdot q = q(2 + q)
\]

or adding and subtracting unity,

\[
\frac{M_2}{M_1} = 1 + 2q + q^2 - 1 = (1 + q)^2 - 1. \tag{22}
\]

Thus

\[
\frac{M_2}{M_1} + 1 = (1 + q)^2. \tag{22a}
\]

Equation (22) and Table 1 were used to compute the figures listed in Table 3. This table shows that an impractically large quantity of propellant is needed in order to accelerate a rocket to some very large velocity and then bring it to a standstill.

**Table 3.**

<table>
<thead>
<tr>
<th>(V_1), m/sec</th>
<th>(\frac{M_2}{M_1})</th>
<th>(\frac{M_3}{M_1})</th>
<th>(V_2), m/sec</th>
<th>(\frac{M_2}{M_1})</th>
<th>(\frac{M_3}{M_1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>543</td>
<td>0.1</td>
<td>0.21</td>
<td>11,800</td>
<td>7</td>
<td>63</td>
</tr>
<tr>
<td>1,037</td>
<td>0.2</td>
<td>0.44</td>
<td>12,500</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>1,493</td>
<td>0.3</td>
<td>0.69</td>
<td>13,100</td>
<td>9</td>
<td>99</td>
</tr>
<tr>
<td>1,915</td>
<td>0.4</td>
<td>0.96</td>
<td>13,650</td>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>2,398</td>
<td>0.5</td>
<td>1.25</td>
<td>17,160</td>
<td>19</td>
<td>399</td>
</tr>
<tr>
<td>3,920</td>
<td>1</td>
<td>3</td>
<td>17,320</td>
<td>20</td>
<td>440</td>
</tr>
<tr>
<td>6,260</td>
<td>2</td>
<td>8</td>
<td>19,560</td>
<td>30</td>
<td>960</td>
</tr>
<tr>
<td>7,880</td>
<td>3</td>
<td>15</td>
<td>22,400</td>
<td>50</td>
<td>2,600</td>
</tr>
<tr>
<td>9,170</td>
<td>4</td>
<td>24</td>
<td>28,280</td>
<td>100</td>
<td>10,200</td>
</tr>
<tr>
<td>10,100</td>
<td>5</td>
<td>35</td>
<td>30,038</td>
<td>193</td>
<td>37,248</td>
</tr>
<tr>
<td>11,100</td>
<td>6</td>
<td>48</td>
<td>Infinite</td>
<td>Infinite</td>
<td>Infinite</td>
</tr>
</tbody>
</table>
Note that equations (22a) and (6) also yield the numerical results listed in Table 3:

\[
\frac{M_t}{M_i} - 1 = e^{2v},
\]

5. **BRAKING OF A ROCKET IN A GRAVITATIONAL FIELD**

The effect of the propellant in a gravitational field is the same when the rocket ascends from the ground to a certain altitude, moving against the force of gravity and spending all its momentum, or when the rocket falls from this ultimate altitude to the ground and the propellant checks its fall.

Braking in a gravitational field requires a larger expenditure of propellant than in free space, so that the \( q \) in (21) and (22) is larger. We will denote this new value by \( q_1 \).

Since the work done by the propellant is proportional to its mass, we have

\[
\frac{T_s}{q} = \frac{M_t}{M_s} = \frac{M_t/M_i}{M_s/M_i} = q = (by 18) = 1 - \frac{p}{q_1},
\]

whence

\[
q_1 = q \left( \frac{p}{p - \bar{g}} \right).
\]  

(23)

Inserting this expression in (22), we find

\[
\frac{M_t}{M_i} = (1 + q_1)^2 - 1 = \left( 1 + \frac{pq}{p - \bar{g}} \right)^2 - 1.
\]  

(24)

Here \( M_i \) is the quantity of propellant needed to lift the rocket from a given point and to bring it back to that point with zero velocity, in a gravitational field.

Taking \( \frac{p}{\bar{g}} = 10 \), we compute Table 4.

**TABLE 4.**

<table>
<thead>
<tr>
<th>( \frac{M_t}{M_i} = q )</th>
<th>( \frac{M_s}{M_i} )</th>
<th>( \frac{M_t}{M_i} = q )</th>
<th>( \frac{M_s}{M_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.235</td>
<td>3</td>
<td>17.78</td>
</tr>
<tr>
<td>0.3</td>
<td>0.778</td>
<td>4</td>
<td>28.64</td>
</tr>
<tr>
<td>0.5</td>
<td>1.420</td>
<td>5</td>
<td>41.98</td>
</tr>
<tr>
<td>1.0</td>
<td>4.457</td>
<td>6</td>
<td>57.78</td>
</tr>
<tr>
<td>2</td>
<td>9.383</td>
<td>7</td>
<td>76.05</td>
</tr>
</tbody>
</table>

6. **HORIZONTAL FLIGHT OF A ROCKET**

Although vertical flight seems to be more advantageous, as the rocket crosses the atmosphere faster and ascends to higher altitudes, we should
remember that the work needed to overcome the air drag is negligible compared to the total work done by the propellant, whereas oblique or horizontal motion will enable us to set up a permanent observatory outside the atmosphere which will orbit indefinitely around the earth like the moon. Moreover, in oblique flight, the utilization of the propellant energy is much higher than in vertical motion.

Let us first consider the case of horizontal motion. The rocket moves under the action of two forces: 1) gravitation, 2) recoil. The resultant of these two forces should be horizontal. Let \( R \) be the horizontal acceleration and \( p \) the acceleration caused by propellant explosion; \( g \) is the gravitational acceleration (Figure 32). We then have

\[
R = \sqrt{p^2 - g^2}. \tag{25}
\]

The kinetic energy acquired by the rocket in time \( t \) is

\[
\frac{R}{2} t^2 = \frac{R^2}{2g} t^2 = \frac{p^2 - g^2}{2g} t^2. \tag{26}
\]

The work of the propellant under free-space conditions is

\[
\frac{p}{2} t^2 = \frac{p^2}{2g} t^2. \tag{27}
\]

Dividing (26) through by (27), we find the utilization in horizontal flight:

\[
1 - \left(\frac{g}{p}\right)^2. \tag{28}
\]

Air drag is ignored. We see from (28) that the loss of work relative to free-space conditions is characterized by the ratio \( \left(\frac{g}{p}\right)^2 \) and it is substantially less than in vertical motion. Thus, for \( \frac{g}{p} = \frac{1}{10} \), the loss is \( \left(\frac{g}{p}\right)^2 = \frac{1}{100} \), whereas in vertical motion it is \( \frac{g}{p} = \frac{1}{10} \).

Table 5 lists the losses for various values of \( \frac{p}{g} \) and the corresponding angles between the directions of \( p \) and \( R \).

<table>
<thead>
<tr>
<th>( \frac{p}{g} )</th>
<th>Loss</th>
<th>( \sin \theta )</th>
<th>( \theta^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>1:4</td>
<td>1:2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>1:9</td>
<td>1:3</td>
<td>19.5</td>
</tr>
<tr>
<td>4</td>
<td>1:16</td>
<td>1:4</td>
<td>14.5</td>
</tr>
<tr>
<td>5</td>
<td>1:25</td>
<td>1:5</td>
<td>11.5</td>
</tr>
<tr>
<td>10</td>
<td>1:100</td>
<td>1:10</td>
<td>5.7</td>
</tr>
<tr>
<td>100</td>
<td>1:10,000</td>
<td>1:100</td>
<td>0.57</td>
</tr>
</tbody>
</table>

58
7. OBLIQUE FLIGHT OF A ROCKET

The horizontal flight of a rocket is disadvantageous because it involves a substantially longer path through air and proportionately larger losses of energy in overcoming air drag. Let \( \alpha \) be the angle of inclination of the trajectory \( R \) to the vertical (Figure 33). This angle is greater than 90°. We have from the drawing

\[
R = \sqrt{p^2 + g^2 + 2pg \cos \gamma}. \tag{29}
\]

where

\[
\gamma = 180° - (\alpha + \beta).
\]

Moreover

\[
\sin \alpha : \sin \beta : \sin \gamma = p : g : R \quad \text{and} \quad \cos \alpha = \frac{R_1 + g^2 - p^2}{2g^2}.
\]

The kinetic energy is expressed by relation (26), where \( R \) is defined by (29).

The vertical acceleration is

\[
R_1 = R \cos (180° - \alpha) = -R \cos \alpha. \tag{30}
\]

The work needed to lift the rocket is

\[
\frac{R^2}{2} t^2 = -\frac{R^2}{2} \cos \alpha, \tag{31}
\]

where \( t \) is the time for the entire propellant reserve to burn out. The total work acquired by the rocket in a gravitational field is

\[
\frac{R^2}{2g} t^2 - \frac{R^2}{2} \cos \alpha = \frac{R_1 t}{2} \left( \frac{R}{g} - \cos \alpha \right). \tag{32}
\]

Here the work required to lift the rocket to unit height with acceleration \( \kappa \) is taken as 1.

The work in free space is, by (27),

\[
\frac{\kappa^2}{2g} t^2
\]

(\( t \) is independent of the gravitational forces).

The utilization of the propellant energy in a gravitational field relative to free-space utilization is obtained by dividing (32) through by (27):

\[
\frac{\kappa^2}{2g} t^2 \left( \frac{R}{g} - \cos \alpha \right) = \frac{p^2}{2} \left( \frac{R}{p} - \frac{g}{p} \cos \alpha \right) \tag{33}
\]

Inserting \( R \) from (29), we find

\[
1 + \frac{p^2}{p^2} + 2 \cos \gamma \frac{g}{p} - \cos \alpha \frac{g}{p} \sqrt{1 + \frac{k^2}{p^2} + \cos \gamma \frac{k}{p}}. \tag{34}
\]
Equations (18) and (28) are particular cases of (34).

**Example.** A rocket flies at an angle of 14.5° to the horizon. The sine of this angle is 0.25; the air drag is thus 4 times the air drag in vertical flight, being approximately proportional to the sine of the angle $\alpha - 90^\circ$ that the trajectory makes with the horizon.

We have

$$\alpha = 90 + 14.5^\circ = 104.5^\circ; \cos \alpha = 0.25; \sin \beta = -\sin \frac{\beta}{\rho};$$

if

$$\frac{\beta}{\rho} = 0.1,$$

then

$$\sin \beta = 0.0968; \beta = 5.7^\circ; \gamma = 110^\circ; \cos \gamma = -0.342.$$

Equation (34) gives the utilization, 0.966. The losses are 0.034 or about 3.4%. This loss is $\frac{1}{3}$ of the corresponding figure for vertical flight. The result is quite adequate, especially if we recall that the air drag in oblique motion (14.5°) does not exceed 1% of the total work needed to launch the projectile into space.

Table 4 lists the utilizations and the losses for variously inclined trajectories. The first column gives the inclination to the horizon (Figure 33). $\beta$ is the angle between the trajectory and the direction of ejection of the propellant gases.

<table>
<thead>
<tr>
<th>$\alpha - 90^\circ$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\alpha + \beta$</th>
<th>Utilization</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
<td>$5.7^\circ$</td>
<td>$95.7^\circ$</td>
<td>0.9900</td>
<td>1:100</td>
</tr>
<tr>
<td>2</td>
<td>92</td>
<td>$5.7^\circ$</td>
<td>$97.7^\circ$</td>
<td>0.9850</td>
<td>1:72</td>
</tr>
<tr>
<td>5</td>
<td>95</td>
<td>$5.7^\circ$</td>
<td>$100.7^\circ$</td>
<td>0.9800</td>
<td>1:53</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>$5.7^\circ$</td>
<td>$105.7^\circ$</td>
<td>0.9791</td>
<td>1:37</td>
</tr>
<tr>
<td>15</td>
<td>105</td>
<td>$5.7^\circ$</td>
<td>$110.7^\circ$</td>
<td>0.9651</td>
<td>1:29</td>
</tr>
<tr>
<td>20</td>
<td>110</td>
<td>$5.7^\circ$</td>
<td>$115.7^\circ$</td>
<td>0.9573</td>
<td>1:23.4</td>
</tr>
<tr>
<td>30</td>
<td>120</td>
<td>$5^\circ$</td>
<td>125</td>
<td>0.9426</td>
<td>1:17.4</td>
</tr>
<tr>
<td>40</td>
<td>130</td>
<td>$4.1^\circ$</td>
<td>$134.1^\circ$</td>
<td>0.9300</td>
<td>1:14.3</td>
</tr>
<tr>
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<td>4</td>
<td>139</td>
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<td>1:13.3</td>
</tr>
<tr>
<td>90</td>
<td>180</td>
<td>0</td>
<td>180</td>
<td>0.9000</td>
<td>1:10</td>
</tr>
</tbody>
</table>

For very small inclination angles $\alpha - 90^\circ$, relation (34) can be simplified by substituting the arc values for the trigonometric functions and making some other simplifications.

We thus obtain the following expression for the loss of work:

$$x^2 + \beta \cdot x \left(1 - \frac{\beta}{2}\right) + \beta^2 \cdot x^2 \left(x - \frac{1}{2}\right).$$

5818

60
Here $\delta$ is the inclination of the trajectory ($\alpha - 90^\circ$) in radian measure and $x = \frac{r}{p}$. Omitting small quantities of higher orders, we find

$$x^2 + \delta x - \left(\frac{\delta}{p}\right)^2 + \delta \frac{\delta}{p}.$$

Let $\delta = 0.02 N$, where 0.02 is the radian measure of an angle close to one degree ($11^{1/2}_7$ to be precise) and $N$ its measure in degrees. The loss of work is then expressed by

$$\frac{\delta}{p^2} + 0.02 \frac{\delta}{p} N.$$

Taking, as before, $\frac{\delta}{p} = 0.1$, we compute Table 7. For angles less than $10^\circ$, the results are close to those in Table 6.

### Table 7.

<table>
<thead>
<tr>
<th>$\theta^\circ$</th>
<th>Loss</th>
<th>$\theta^\circ$</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1:100</td>
<td>4</td>
<td>1:55</td>
</tr>
<tr>
<td>0.5</td>
<td>1:91</td>
<td>5</td>
<td>1:50</td>
</tr>
<tr>
<td>1</td>
<td>1:83</td>
<td>6</td>
<td>1:45</td>
</tr>
<tr>
<td>2</td>
<td>1:70</td>
<td>10</td>
<td>1:33</td>
</tr>
<tr>
<td>3</td>
<td>1:60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. LAUNCH FROM A PLANET

Launch from the earth. In his "Beyond the Earth" (Vne Zemli) (p. 18), K. Tsiolkovskii describes the launching of a rocket in the following words: "The rocket first flies parallel to the equatorial plane making an angle of $25^\circ$ with the horizon in the sense of the earth's diurnal rotation. During the first 10 sec, its velocity is swiftly accelerated to 500 m/sec; during the rest of its flight through the air, the acceleration is much slower, but continuous as long as the propellant burns. Having traversed the air mass, the rocket will resume rapid acceleration, while the direction of flight will gradually change. At an altitude of 1,000 km, the rocket will start tracing a circular trajectory, and its velocity will be sufficient to maintain it in a circular orbit around the earth, without allowing it to drop any closer to the surface. The orbiting velocity around the earth is 7.5 km/sec."

On p. 23, the author describes the impressions of the eyewitnesses who observed the launching of the rocket from the ground. "They saw how the rocket lifted off the ground and swiftly moved into space, in an inclined direction. Many of them recoiled with fear. There was a deafening noise, but it rapidly abated as the rocket moved farther and farther. The rocket flew eastward, following the earth in its diurnal axial rotation. At the same time, the rocket continued climbing upward, and after 10 sec it was at a distance of 5 km from the observers, moving with a speed of 1,000 m/sec (500 m/sec faster). It was discernible with difficulty through
powerful binoculars at this distance, mainly because it began to glow as a result of air friction. In fact, the rocket disappeared almost instantaneously from the eyes of the observers. Suddenly a rolling thunder was heard. At first it increased in strength, and then diminished. The rolling thunder persisted, although the rocket was no longer visible. The crowds stared in surprise, but there were no thunderstorm clouds in the sky. The noise was produced by the rocket which cut through the air creating a thunder-like sound wave.

Launch from the moon. Tsiolkovskii gives the following description in his "Beyond the Earth" (pp. 91-92). "A smooth area was selected—a part of a mountain slope rising at an angle of 10–20 degrees. The rocket was mounted on this inclined plot. The crew were secured inside and the propellant was touched off. At first they moved along the mountain slope, and then lifted into space, near the moon. They climbed and accelerated, reaching a final velocity of 1,600 m/sec. The propellant explosion was then switched off. They were orbiting around the moon at an altitude of 265 km from its surface."

After a few hours in lunar orbit, the propellant was again turned on, their velocity was increased to 2.5 km/sec, the propellant was turned off, and they continued their journey from the moon to an earth orbit.

Launch from an asteroid required progressively smaller propellant reserves as the asteroid mass becomes smaller.

9. LANDING ON A PLANET

Landing on the earth. After a trip to outer planets, Tsiolkovskii's rocket returns to the earth ("Beyond the Earth," p. 111), describing a spiral trajectory around the sun and gradually drawing closer to the earth's orbit. When the rocket was at a distance of 65 million km from the earth's orbit, braking began from a velocity of 25 km/sec. The solar attraction, however, increased this velocity and on reaching the earth's orbit the velocity of the rocket matched that of the earth. The earth's attraction began to be felt. Braking was initiated by firing retro-jets. When the earth was near, the crew climbed into liquid tanks. On re-entry into the atmosphere, the outer protective shell of the rocket became incandescent, but the rocket speed was then moderate, and it continued to decrease as the rocket approached the water surface of the ocean. Another powerful braking burst, and the rocket almost came to standstill. A slight splash and the rocket floated like a ship.

Landing on the moon. The pull of the moon attracted the rocket from its lunar orbit until the relative velocity of the rocket reached 2 km/sec. When the rocket was at a distance of 2,000 km from the moon, braking began and the rocket was brought to a standstill at an altitude of 3 km above the lunar surface. It was then accelerated to 100 m/sec toward the moon. At an altitude of 500 m, it was again braked and finally the rocket landed with a scarcely perceptible jolt, turned, rested on its landing gear and rolled over a few tens of meters on the ground coming to standstill ("Beyond the Earth," p. 88).
Landing on an asteroid ("Beyond the Earth," p. 103). When the asteroid was sighted, the rocket was maneuvered toward it by judicious firing of the jets. At a sufficient proximity to the asteroid, the rocket was slowed down by retro-jets and gradually brought to a distance of a few tens of meters from the asteroid, where the rocket remained in a state of relative rest. Finally, the rocket landed due to the combined action of gravitational attraction and retro-jet firing.

There are many other factors to be considered: the work of gravitational forces, air drag, the behavior of human beings in airless space, navigation of the rocket in the atmosphere and in outer space.

We have so far dealt with Tsiolkovskii's original theory as it was published in 1903. The development of the theory was continued in 1911-1912 in "Vestnik Vozdushnoplavaniya" (Aeronautical Journal), and the results are presented in the following sections.

10. WORK OF GRAVITATION FOR ESCAPE FROM A PLANET

When a unit mass is moved from the surface of a planet of radius \( r \) to an altitude \( h \), the work done \( T \) is given by the relation

\[
T = \frac{g}{g_1} r_1 \left(1 - \frac{r_1}{r_1 + h}\right)
\]

(35)

Here \( g \) is the gravitational acceleration at the surface of the planet and \( g_1 \) is the Earth's gravitational acceleration.

If

\[
h = \infty, \text{ then } T = \frac{g}{g_1} \cdot r_1,
\]

(36)

i.e., the work needed to move a unit mass from the surface of a planet to infinity is equal to the work needed to lift the same mass to an altitude of one planetary radius above the surface, assuming constant gravitational acceleration to this altitude.

For uniform-density planets, the gravitational acceleration at the surface is proportional to the radius of the planet. It is expressed by the ratio of the planetary radius \( r_1 \) to the Earth's radius \( r_2 \):

\[
\frac{g}{g_1} = \frac{r_1}{r_2}; \text{ } T = \frac{r_1}{r_2} \cdot r_1 = \frac{r_1^2}{r_2}
\]

This indicates that the limit work \( T_1 \) rapidly diminishes as the planetary radius \( r_1 \) decreases.

For the Earth, \( T_1 = 6,366,000 \text{ kg-m} \), which is less than the energy of 1.5 kg of petroleum.

Thus, about 70 kg of petroleum will be needed to lift a 70-kg person.

For the moons of Mars (10 km in diameter), \( T_1 = 16 \text{ kg-m} \).

For the moon, \( T_1 = \frac{T_{1, \text{ earth}}}{22} \).
For the asteroid Vesta, \( T_i = \frac{T_i \text{earth}}{1000} \) (Vesta's diameter is 500 km).

For any planet, \( T_i = \frac{h}{h + r_i} = \frac{r_i}{1 + \frac{r_i}{h}} \).

Using this formula, we computed the figures listed in Table 8.

| Altitude in units of planetary radius \( \frac{h}{r_i} \) | \( \frac{1}{10} \) | \( \frac{1}{5} \) | \( \frac{1}{3} \) | \( \frac{1}{2} \) | \( 1 \) | 2 | 3 | 9 | 99 | Infinite \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of work to work needed to move to infinity ( \frac{T}{T_i} )</td>
<td>( \frac{1}{11} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( 2 )</td>
<td>( 3 )</td>
<td>( \frac{3}{4} )</td>
<td>( \frac{9}{10} )</td>
<td>( \frac{99}{100} )</td>
<td>1</td>
</tr>
</tbody>
</table>

### TABLE 8.

#### 72 11. ESCAPE VELOCITY

The velocity \( V_i \) needed to raise a rocket to an altitude \( h \) and to attain a terminal velocity \( V \) at this altitude is

\[
V_i = \sqrt{V^2 + \frac{2g r_i h}{r_i + h}}
\]  

(37)

If \( V = 0 \), i.e., the object moves until it is stopped by gravitational forces, we have

\[
V_i = \sqrt{\frac{2g r_i h}{r_i + h}}
\]  

(38)

If \( h = \infty \), we have

\[
V_i = \sqrt{2g r_i}
\]  

(39)

Thus, for the earth \( V_i = 11,170 \) m/sec, 
for the moon \( V_i = 2,373 \) m/sec 
for an asteroid \((D = 6^{1/3} \text{ km})\) \( V_i = 5.7 \) m/sec. 
For planets having the same density as the earth 

\[
V_i = r_i \sqrt{\frac{2g}{r_i}}
\]  

(40)

where \( g \) and \( r_i \) refer to the earth.

For Vesta \((D = 400 \text{ km})\), \( V_i = 324 \) m/sec.

To achieve permanent orbiting around a planet, only half the escape work is needed. The velocity for placing a satellite in orbit is a factor of \( \sqrt{2} = 1.41 \) times less than the escape velocity.
12. FLIGHT TIME

The time $t$ for an initially stationary object to fall on a planet or the sun, assuming that their mass is concentrated at a single point, is

$$t = \frac{2}{\sqrt{g}} \sqrt{\frac{r}{r_1^2}} \left[ V^2 \sqrt{\frac{r}{r_1}} - 1 + \arcsin \sqrt{\frac{r}{r_1}} \right],$$

(41)

where $r_1$ is the distance from which the fall begins, $r$ is the distance traversed during the fall, $r_1$ is the radius of the planet, $g$ is the gravitational acceleration at the surface of the planet.

A similar relation expresses the time to ascend from altitude $r_2 - r$ to $r_2$, when the object is being slowed down by the gravitational pull.

If $r = r_2$, we have

$$t = \frac{\pi}{2} \cdot \frac{r_2}{r_1} \sqrt{\frac{r_2}{2g}}.$$

(42)

Although $t$ is the time to fall to the center of mass of the planet, the same relation is applicable under ordinary conditions to determine the time of fall to the surface of the planet or the time of ascent from its surface.

On the other hand, the time to complete one circuit around the planet is

$$t_c = 2\pi \frac{r_2}{r_1} \sqrt{\frac{r_2}{g}},$$

(43)

73 where $r$ is the radius of the planet, $g$ is the gravitational acceleration on its surface, $r$ is the distance of the object from the center.

Comparing (42) and (43) we find

$$t_c : t = 4 \sqrt{2} - 5.657,$$

(44)

i.e., the ratio of the time to complete one circuit of the satellite orbit to the time it takes the satellite to fall to the center of the planet (assuming a point mass concentration) is constant, being equal to 5.66.

Thus the moon would take 4.8 days to fall on the earth and the earth would take 4.25 days to fall on the sun.

Conversely, a rocket launched from the earth would cruise by inertia for 4.8 days until it stops at the moon's orbit.

13. AIR DRAG

Let us find the work done by a rocket against atmospheric drag in uniformly accelerated motion. We have to take into consideration the variable air density at various altitudes.

Tsiolkovskii expressed the air density by the relation*

$$d = d_1 \left(1 - \frac{d_1 A}{2(\alpha + 1)}\right)^{\frac{A+1}{2}},$$

(45)

* For the derivation see the journal "Vozdushnoe Plavanie" (The Aeronaut), No.3, p.66, 1905.
where

\[ A = \frac{d_i \cdot M \cdot T \cdot C}{f}, \]

Here \( d_i \) is the air density at sea level, \( (d_i = 1.0013) \), \( h \) is the altitude, \( f \) is the air pressure at sea level \( (f = 103.33 \text{ kg/dm}^2) \), \( M \) is the mechanical equivalent of heat \( (M = 4,240 \text{ kg/dm}) \), \( C \) is the specific heat of air at constant volume \( (0.169) \), \( T \) is the absolute zero \( (-273^\circ \text{C}) \). Inserting these numerical values we find \( A = 2.441 \) and

\[ d = d_i \left(1 - \frac{h}{h_i}\right)^a, \quad (46) \]

where

\[ a = 2A + 1 = 5.884 \text{ and } h_i = \frac{2(A + 1)}{d_i} f = 54.54 \text{ km}. \]

This is the limiting theoretical height of the atmosphere which is obtained from (45) for \( d = 0 \). (At this height we have in fact \( d = 0.001 \), even if we assume constant temperature, equal to the ground temperature.)

The differential of the air-drag work is

\[ dT = F \cdot dh, \quad (47) \]

where \( F \) is the air-drag force:

\[ F = \frac{K \cdot S \cdot d \cdot V^2}{2g \cdot u^2}, \quad (48) \]

74 \( K \) is the air-drag coefficient \( (1.4, \text{ according to Langley}) \), \( S \) is the frontal cross section of the rocket, \( d \) is the air density at the corresponding altitude, \( V \) is the velocity of the rocket, \( g \) is the gravitational acceleration of the earth \( (9.8) \), \( u \) is the rocket shape coefficient (assumed constant).

Since the air drag is small compared to the propellant reaction forces (about 1%), we may take

\[ V = \sqrt{2(p - g)} \cdot h, \quad (49) \]

where \( p - g \) is the true acceleration of the projectile.

This simplification, leading to higher velocity values, will increase the work of air drag and thus offset the error which was introduced when we took 54.54 \text{ km} for the scale height of the atmosphere.

From (46), (47), (48), and (49), we have

\[ dT = b \left(1 - \frac{h}{h_i}\right)^a h \cdot dh, \quad (50) \]

where

\[ b = \frac{K \cdot d_i \cdot S(p - g)}{u \cdot g}, \quad (51) \]
Integration by parts gives

\[ T = b \left\{ \frac{h_i^2}{(a+1)(a+2)} \left[ 1 - \left( \frac{1 - \frac{h_i^2}{h_i^2} + 1}{a+1} \right) \right] \right\} \]  

(52)

Taking \( h = h_i \), we find the total work of air drag

\[ T_i = b \cdot \frac{h_i^2}{(a+1)(a+2)} \]  

(53)

Let

\[ K = 1.4; \ d = 0.0013; \ S = 2 \text{ m}^2; \ \frac{P}{g} = 10; \ g = 9.8 \text{ m/sec}^2; \ u = 100. \]

Then

\[ b = 0.0003276; \ a = 5.88; \ h_i = 54.54 \text{ m}; \ T_i = 17,975 \text{ ton/m}. \]

The work of 1 ton of an explosive hydrogen-oxygen mixture yielding 1 ton of water is 1,600,000 ton-m.

If the rocket with its payload and crew weighs 1 ton, and the quantity of the propellant is 6 ton, the rocket will carry potential energy of 9,600,000 ton-m. More than half of this potential energy is converted into kinetic energy, i.e., somewhat more than 4,800,000 ton-m. The work of air drag is thus only about \( \frac{1}{100} \) of the work of gravity. We can find the same result by directly dividing the air drag work (17,975) by the total work of gravity \((6,336,000)\), which gives about \( \frac{1}{35} \).

Table 9 lists, under the given conditions, the time (in seconds) from the vertical launch, the corresponding velocity of the rocket in m/sec, the ascent height in m, and the air density. The air density at sea level is taken as unity, and the temperature is assumed to decrease uniformly at a rate of 5°C per 1,000 m of altitude (see "Vozdukhoplavatel'" (The Aeronaut), No. 3, p. 65, 1905).

The work of air drag is nearly \( \frac{1}{35} \) of the total loss of work due to gravitational attraction in vertical ascent. It is therefore advisable to choose an inclined trajectory: this policy will increase by a certain factor a small quantity — the air drag — while decreasing a much larger quantity — loss of energy due to gravitation. The work of air drag may be taken approximately equal to cosec² \( (90°-\alpha) \), here \( (90°-\alpha) \) is the angle of inclination of the trajectory to the horizon. This assumption is valid up to a certain angle.

Table 10 lists the angles of inclination of the trajectory to the horizon, the losses of kinetic energy due to gravitation and due to air drag \( (U = 100) \), and the total losses.

For \( U =100 \), the minimum losses are observed for \( 90°-\alpha = 10-15° \).

The losses amount to 0.07 of the entire energy of the propellant in free space, or to about 7%.
(75) TABLE 9.

<table>
<thead>
<tr>
<th>Time of ascent ( t ), sec</th>
<th>Velocity ( V ), m/sec</th>
<th>Height ( h ), m</th>
<th>Relative air density, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>180</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>270</td>
<td>405</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>450</td>
<td>1,125</td>
<td>1:1.13</td>
</tr>
<tr>
<td>7</td>
<td>630</td>
<td>2,205</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>900</td>
<td>4,500</td>
<td>1:1.633</td>
</tr>
<tr>
<td>15</td>
<td>1,350</td>
<td>10,125</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>1,800</td>
<td>18,000</td>
<td>1:10.83</td>
</tr>
<tr>
<td>30</td>
<td>2,700</td>
<td>40,500</td>
<td>1:2828</td>
</tr>
<tr>
<td>40</td>
<td>3,600</td>
<td>72,000</td>
<td>Close to zero</td>
</tr>
<tr>
<td>50</td>
<td>4,500</td>
<td>112,500</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>6,300</td>
<td>220,500</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>9,000</td>
<td>450,000</td>
<td></td>
</tr>
<tr>
<td>113</td>
<td>9,900</td>
<td>574,000</td>
<td></td>
</tr>
</tbody>
</table>

* A table similar to Table 9 was computed for \( \frac{p}{d} = 3 \) in Tsiolkovskii's "Beyond the Earth" (p. 4). This table is reproduced here as Table 9a.

(75) TABLE 9a.

<table>
<thead>
<tr>
<th>Time of ascent ( t ), sec</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ( V' ), m/sec</td>
<td>20</td>
<td>40</td>
<td>200</td>
<td>1,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Height ( h ), m</td>
<td>10</td>
<td>40</td>
<td>1,000</td>
<td>25,000</td>
<td>100,000</td>
</tr>
</tbody>
</table>

TABLE 10.

<table>
<thead>
<tr>
<th>Angle of inclination of the trajectory to the horizon ( 90^\circ - \alpha )</th>
<th>( \sin (90^\circ - \alpha) )</th>
<th>( \csc (90^\circ - \alpha) )</th>
<th>Energy losses due to gravitation (See Table 6)</th>
<th>Energy losses due to air drag ( U = 100 )</th>
<th>Energy losses due to gravitation and air drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>\infty</td>
<td>0.010</td>
<td>\infty</td>
<td>2.47</td>
</tr>
<tr>
<td>2</td>
<td>0.0349</td>
<td>28.7</td>
<td>0.014</td>
<td>824</td>
<td>0.395</td>
</tr>
<tr>
<td>5</td>
<td>0.0972</td>
<td>11.5</td>
<td>0.020</td>
<td>132</td>
<td>0.099</td>
</tr>
<tr>
<td>10</td>
<td>0.174</td>
<td>5.75</td>
<td>0.027</td>
<td>33.1</td>
<td>0.0477</td>
</tr>
<tr>
<td>15</td>
<td>0.259</td>
<td>3.86</td>
<td>0.035</td>
<td>14.9</td>
<td>0.0255</td>
</tr>
<tr>
<td>20</td>
<td>0.342</td>
<td>2.82</td>
<td>0.045</td>
<td>8.53</td>
<td>0.0120</td>
</tr>
<tr>
<td>30</td>
<td>0.500</td>
<td>2.00</td>
<td>0.057</td>
<td>4.00</td>
<td>0.0159</td>
</tr>
<tr>
<td>40</td>
<td>0.643</td>
<td>1.56</td>
<td>0.070</td>
<td>2.43</td>
<td>0.0073</td>
</tr>
<tr>
<td>45</td>
<td>0.707</td>
<td>1.41</td>
<td>0.075</td>
<td>1.99</td>
<td>0.0059</td>
</tr>
<tr>
<td>90</td>
<td>1.000</td>
<td>1.00</td>
<td>0.100</td>
<td>1.00</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

68
In free space, with the propellant reserves amounting to six times the mass of the rest of the rocket, 0.63 of the latent propellant energy is utilized. Discounting 7% on account of the losses, we conclude that 59% of the total chemical energy of the propellant may be utilized in oblique motion.

The work of air drag may be reduced by a certain factor if the rocket is launched from the peak of a high mountain or if it is first raised by a dirigible or an aircraft to a high altitude, where the air is tenuous.

Thus, launching the rocket from an altitude of 5 km will halve the air drag losses, and from 10 km will quarter the losses.

14. FLIGHT TRAJECTORIES
("Vestnik Vozdihxoplavaniya" (Aeronautical Journal), No. 2 and 3, 1912)

The flight trajectory will change depending on the direction and the site of launching, the magnitude and the direction of the initial velocity, and the quantity of propellant (the ratio of propellant mass to the remaining mass of the rocket, $M_p/M_i$). Figures 34–35 and Table 11 illustrate the different cases which are possible in practice. The drawings and the table are based on the explicit instructions of Tsiolkovskii published in the paper referred to. The trajectories are identified by numbers which correspond to the numbered lines of the table (Tsiolkovskii's paper contains no drawings and no table).

The trajectories were computed ignoring air drag. If air drag is taken into consideration, the rocket velocities should be higher than those indicated in Table 11 and in the drawings, as otherwise the rocket will either fall to the ground or describe a different curve.

The table gives only the ratios $M_p/M_i$ required to launch the rocket. To bring the rocket back safely, the propellant mass $M_p$ should be doubled for small distances from the earth, trebled for larger distances, multiplied by 4 for still larger distances, etc. (see (24)). In mid-flight, the rocket jets can be fired once or in several successive bursts to alter the initial trajectory. Thus, an elliptical trajectory having a point of contact with the ground (No. 3) may be altered into another elliptical trajectory (No. 13) which will place the rocket in a satellite orbit around the earth or even into a parabolic trajectory (No. 14). Finally, by successively firing the jets, the rocket can be made to describe a spiral path: if the bursts increase the rocket velocity, an unwinding spiral will be traced, whereas if the bursts reduce the velocity, a shrinking spiral will be obtained (No. 15). The gravitational forces have a reverse effect on velocity.

If a rocket has a velocity of 30,000 m/sec against the sense of the earth's orbital motion, it will fall on the sun (Figure 35, No. 25). If a rocket moving in a circular orbit around the earth is accelerated to $V = 11,170$ m/sec, it will continue orbiting around the sun (No. 20). If the rocket is accelerated by firing some of the propellant, it will describe an unwinding spiral and may thus reach the outer planets. If the rocket is decelerated, it will trace a contracting spiral and possibly touch on the inner planets. The gravitational forces tend to change the velocities in the opposite direction.
FIGURE 34. Different flight trajectories
<table>
<thead>
<tr>
<th>No.</th>
<th>Direction of launch</th>
<th>$M_e/M_i$</th>
<th>$V$, m/sec</th>
<th>Flight trajectory</th>
<th>Outcome</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vertical from the pole (attraction of celestial bodies ignored)</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Straight line</td>
<td>Falling back to earth</td>
<td>34 (No.1)</td>
</tr>
<tr>
<td>2</td>
<td>Same</td>
<td>8</td>
<td>11,170</td>
<td>Same</td>
<td>Escape to infinity</td>
<td>34 (No.2)</td>
</tr>
<tr>
<td>3</td>
<td>Vertical, not from the pole</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Ellipse</td>
<td>Falling back to earth</td>
<td>34 (No.3)</td>
</tr>
<tr>
<td>4</td>
<td>Same</td>
<td>8</td>
<td>11,170</td>
<td>Parabola</td>
<td>Escape to infinity</td>
<td>34 (No.4)</td>
</tr>
<tr>
<td>5</td>
<td>Same</td>
<td>&gt;8</td>
<td>&gt;11,170</td>
<td>Hyperbola</td>
<td>Same</td>
<td>34 (No.5)</td>
</tr>
<tr>
<td>6</td>
<td>Horizontal</td>
<td>&lt;3–4</td>
<td>&lt;7,904</td>
<td>Ellipse, parabola, or hyperbola</td>
<td>Falling back to earth</td>
<td>34 (No.6)</td>
</tr>
<tr>
<td>7</td>
<td>Same</td>
<td>3–5</td>
<td>$7,904-11,170/\sqrt{2}$</td>
<td>Circle</td>
<td>Earth's satellite in circular orbit</td>
<td>34 (No.7)</td>
</tr>
<tr>
<td>8</td>
<td>Same</td>
<td>&gt;3–4</td>
<td>&gt;7,904</td>
<td>Ellipse</td>
<td>Earth's satellite in elliptical orbit</td>
<td>34 (No.8)</td>
</tr>
<tr>
<td>9</td>
<td>Same</td>
<td>8</td>
<td>11,170</td>
<td>Parabola</td>
<td>Escape to infinity</td>
<td>34 (No.9)</td>
</tr>
<tr>
<td>10</td>
<td>Same</td>
<td>&gt;8</td>
<td>&gt;11,170</td>
<td>Hyperbola</td>
<td>Same</td>
<td>34 (No.10)</td>
</tr>
<tr>
<td>11</td>
<td>Same, from the equator in the sense of axial rotation of the earth</td>
<td>&lt;3–4</td>
<td>$7,904-463=7,441$</td>
<td>Circle</td>
<td>Earth's satellite in circular orbit</td>
<td>34 (No.11)</td>
</tr>
<tr>
<td>12</td>
<td>Inclined at 10–15° to the horizon</td>
<td>&lt;3–4</td>
<td>&lt;7,904</td>
<td>Ellipse</td>
<td>Falling back to earth</td>
<td>34 (No.12)</td>
</tr>
<tr>
<td>13</td>
<td>Same with a boosting burst at apogee</td>
<td>&gt;3–4</td>
<td>&gt;7,904</td>
<td>Ellipse</td>
<td>Earth's satellite in elliptical orbit</td>
<td>34 (No.13)</td>
</tr>
<tr>
<td>14</td>
<td>Same</td>
<td>8</td>
<td>11,170</td>
<td>Parabola</td>
<td>Escape to infinity</td>
<td>34 (No.14)</td>
</tr>
<tr>
<td>15</td>
<td>Inclined with (constant) boosting bursts</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Spiral</td>
<td>Moving toward or away from the earth</td>
<td>34 (No.15)</td>
</tr>
<tr>
<td>No.</td>
<td>Direction of launch</td>
<td>$M_1/M_0$</td>
<td>$V$, m/sec</td>
<td>Flight trajectory</td>
<td>Outcome</td>
<td>Figure No.</td>
</tr>
<tr>
<td>-----</td>
<td>---------------------</td>
<td>----------</td>
<td>------------</td>
<td>------------------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>16</td>
<td>Inclined flight to the moon</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Ellipse</td>
<td>Moon landing $V=2,373$ m/sec</td>
<td>35 (No.16)</td>
</tr>
<tr>
<td>17</td>
<td>Same, the rocket misses the moon</td>
<td>8</td>
<td>11,170</td>
<td>Ellipse</td>
<td>Circumnavigation of the moon and landing on earth</td>
<td>35 (No.17)</td>
</tr>
<tr>
<td>18</td>
<td>Same</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Complex curve</td>
<td>Motion in an orbit around the earth and the moon</td>
<td>35 (No.18)</td>
</tr>
<tr>
<td>19</td>
<td>Same, with a boosting burst</td>
<td>&lt;8</td>
<td>&lt;11,170</td>
<td>Ellipse and circle</td>
<td>Moon's satellite</td>
<td>35 (No.19)</td>
</tr>
</tbody>
</table>

### II. Flight relative to the sun

<table>
<thead>
<tr>
<th>No.</th>
<th>Motion in earth's orbit</th>
<th>$M_1/M_0$</th>
<th>$V$, m/sec</th>
<th>Flight trajectory</th>
<th>Outcome</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Motion in earth's orbit</td>
<td>7-8</td>
<td>11,170</td>
<td>Circle</td>
<td>Sun's satellite</td>
<td>35 (No.20)</td>
</tr>
<tr>
<td>21</td>
<td>Same</td>
<td>&gt;7-8</td>
<td>&gt;11,170</td>
<td>Ellipse</td>
<td>Sun's satellite, touching on outer planets</td>
<td>35 (No.21)</td>
</tr>
<tr>
<td>22</td>
<td>Same</td>
<td>20</td>
<td>16,300</td>
<td>Parabola</td>
<td>Escape from the solar system</td>
<td>35 (No.22)</td>
</tr>
<tr>
<td>23</td>
<td>Same</td>
<td>20</td>
<td>76,300</td>
<td>Hyperbola</td>
<td>Same</td>
<td>35 (No.23)</td>
</tr>
<tr>
<td>24</td>
<td>Same</td>
<td>&lt;7-8</td>
<td>&lt;11,170</td>
<td>Ellipse</td>
<td>Sun's satellite, touching on inner planets</td>
<td>35 (No.23)</td>
</tr>
<tr>
<td>25</td>
<td>Same</td>
<td>200</td>
<td>-30,000</td>
<td>Ellipse</td>
<td>Falling on the sun</td>
<td>35 (No.25)</td>
</tr>
<tr>
<td>26</td>
<td>Same, with continuous boosting</td>
<td>7-8</td>
<td>&gt;11,170</td>
<td>Spiral</td>
<td>Moving toward or away from the sun</td>
<td>35 (No.26)</td>
</tr>
</tbody>
</table>
81 15. TSIOLKOVSKII'S THEOREMS

In 1914, Tsiolkovskii published in Kaluga his booklet entitled "Exploration of Space with Rocket-Propelled Devices" (Issledovanie mirovykh prostranstv reaktivnymi priborami) (Supplement to Parts I and II of a publication bearing the same title). He summarized his work in the form of the following theorems.

Theorem 1. Suppose that the gravitational acceleration remains constant with distance from a planet. Suppose an object is lifted to an altitude equal to one planetary radius. The work done by this object is then equal to the work needed to escape from the gravitational pull of the planet (see above p. 63 and equation (36)).

Theorem 2. In free space, the final velocity of a rocket propelled by gases ejected in a fixed direction is independent of the intensity and the frequency of propellant bursts. It is entirely determined by the quantity of propellant (relative to the mass of the rocket) and the quality and design of the exhaust nozzle (see p. 50 and equation (6)).

Theorem 3. If the quantity of the propellant is equal to the mass of the rocket, almost one half of the energy released by the propellant is transmitted to the rocket (see Table 2 for $\frac{M_p}{M_i} = 1$).
**Theorem 4.** When the mass of the rocket plus the propellant mass increase in a geometrical progression, the rocket velocity increases in an arithmetic progression (see p. 58 and equation (6)).

For example, if the masses increase in the ratio $2, 4, 8, 16, 32, 64, \ldots$ the velocities increase in the ratios $1, 2, 3, 4, 5$, or in numerical values:

<table>
<thead>
<tr>
<th>Propellant mass</th>
<th>1, 3, 7, 15, 31, 63, 127, 256</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, km/sec</td>
<td>$3^{1/2}, 7, 10^{1/2}, 15, 17^{1/2}, 21, 24^{1/2}, 28$</td>
</tr>
</tbody>
</table>

for hydrogen and oxygen

<table>
<thead>
<tr>
<th>Velocity, km/sec</th>
<th>3, 6, 9, 12, 15, 18, 21</th>
</tr>
</thead>
</table>

for benzene or gasoline

**Theorem 5.** During vertical flight of a rocket in a gravitational field, e.g., on earth, part of the energy released by the propellant is lost. The losses increase as the pressure of the ejected gases on the rocket approach the weight of the rocket (see pp. 52–54).
1. INTRODUCTION

In 1926, Tsiolkovskii published in Kaluga his new treatise on interplanetary travel, entitled "Issledovanie mirovykh prostranstv reaktivnymi priborami" (Exploration of Space with Rocket-Propelled Devices). This constituted a new edition, with some alterations and additions, of earlier work, originally published in 1903 and 1911.

Although the author says that the new edition comprises only some alterations and additions to the previous edition, this book dealt with an essentially new range of topics not treated in his earlier publications. Tsiolkovskii, with his characteristic flair, managed to impart a new distinctive flavor to these topics, which prior to 1926 were treated in the work of Oberth, Hohmann, Valier, the Italian artillery school, and other Western authors.

In this chapter, we will present a brief survey of the basic results of this book which have not been discussed so far.

2. TSIOLKOVSKI'S IDEAS REGARDING GUNS

In his 1926 publication, Tsiolkovskii somewhat changed his opinion regarding the application of guns to the launching of projectiles into outer space. He came to the conclusion that guns would eventually be used on a large scale for projectile launching, both in order to assist migration throughout outer space and as an alternative to rocket propulsion. The first stages of the conquest of outer space, i.e., colonization of space near the earth, requires a combination of the gun method with the rocket propulsion method. The projectile acquires velocities of less than 8 m/sec in the gun, and is then further accelerated by its own propellant, like a rocket. Electromagnetic guns have tremendous advantages, as they are simpler to build, more economical, and have an abundant supply of energy along the entire barrel. Tsiolkovskii proceeded to compute a table of flight parameters for a gun projectile, assuming a constant pressure on the projectile inside the gun barrel. Hydrogen was used as the expanding gas, which was heated by special conductors provided in the explosion chamber. The gas density was also assumed constant throughout the gun barrel.

These assumptions lead to the following five fundamental equations:

\[ P = p \cdot n \cdot F_i \]  

(54)
These equations lead to the following four relations:

\[ W = K \cdot g_e \quad \text{(59)} \]
\[ P = (W \cdot M) : g_e \quad \text{(60)} \]
\[ n = P : (F \cdot p) \quad \text{(61)} \]
\[ S = V^2 : (2w) \quad \text{(62)} \]

These relations served as a basis for a table which shows that for \( n = 10^4 \) and \( S = 720 \text{ km} \), we may attain \( V = 380 \text{ km/sec} \).

For \( n = 100, K = 100, S = 144.5 \), we may attain \( V = 17 \text{ km/sec} \).

For \( V = 4 \text{ km/sec} \), we have \( n = 10, K = 10 \), and \( S = 80 \text{ km} \), etc.

Here \( P \) – the pressure on the projectile
\( p \) – the atmospheric pressure (= 10)
\( n \) – pressure in atmospheres
\( F \) – cross-sectional area of the gun barrel
\( W \) – the acceleration of the projectile
\( g_e \) – the earth’s gravitational acceleration,
\( M \) – the mass of the projectile (= 10)
\( V \) – the final velocity in km/sec
\( S \) – the length of the gun in km
\( T_s \) – the time the projectile spends in the barrel
\( K \) – the relative weight of the projectile,
\( D \) – the cross-sectional diameter of the projectile in the barrel.

All measures, except \( S \) and \( V \), are in tons, meters, and seconds.

### Table 12. Flight parameters of a gun projectile

<table>
<thead>
<tr>
<th>( K )</th>
<th>10</th>
<th>100</th>
<th>10</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>1000</th>
<th>1000</th>
<th>1000</th>
<th>1000</th>
<th>10000</th>
<th>10000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td>10^4</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>10</td>
<td>100</td>
<td>10</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( S )</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>14.5</td>
<td>8</td>
<td>80</td>
<td>7.2</td>
<td>7.2</td>
<td>72</td>
<td>72</td>
<td>720</td>
<td>720</td>
<td></td>
</tr>
<tr>
<td>( T_s )</td>
<td>120</td>
<td>120</td>
<td>38</td>
<td>38</td>
<td>8</td>
<td>17</td>
<td>4</td>
<td>40</td>
<td>1.44</td>
<td>3.8</td>
<td>3.8</td>
<td>12</td>
<td>38</td>
<td>20</td>
</tr>
<tr>
<td>( F )</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>( V )</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>8</td>
<td>17</td>
<td>4</td>
<td>4</td>
<td>12</td>
<td>38</td>
<td>38</td>
<td>120</td>
<td>380</td>
<td>8</td>
</tr>
<tr>
<td>( D )</td>
<td>1.13</td>
<td>1.13</td>
<td>3.57</td>
<td>3.57</td>
<td>1.13</td>
<td>1.13</td>
<td>1.13</td>
<td>1.13</td>
<td>1.13</td>
<td>1.13</td>
<td>3.57</td>
<td>3.57</td>
<td>3.57</td>
<td>2.26</td>
</tr>
</tbody>
</table>

84 The length of the gun according to Tsiolkovskii

Let the acceleration of the projectile inside the gun barrel be 1,000 m/sec\(^2\) \((\kappa \cdot 100 = 10 \cdot 100)\). To escape the gravitational pull of the earth, the projectile has to acquire a velocity of 12 km/sec while in the gun barrel. It will attain this velocity in 12 sec. The mean velocity of the projectile inside the
barrel is 6 km/sec. In 12 sec it will have covered 72 km, this being the minimum length of the gun. In all probability, the gun should be 10 times as long, since man cannot sustain accelerations in excess of 10 g, even if immersed in a liquid.

3. GENERAL THEORY OF ROCKET PROPULSION

Consider two cases of rocket propulsion: 1) the substance ejected from the rocket has no intrinsic energy, being driven by some other weightless agent; 2) the ejected substance is driven by its intrinsic energy. Suppose that the propulsion occurs in a medium without gravitation and air drag.

Case 1. Let $M_{i}$ be the mass of the rocket and $V$ the velocity of the rocket. $M_{e}$ is the ejected mass and $V_{e}$ is its velocity.

From the law of conservation of momentum, we have

$$M_{i} \cdot V + M_{e} \cdot V_{e} = 0. \tag{63}$$

The work absorbed by the rocket is

$$T = \frac{M_{i} \cdot V^2}{2}. \tag{64}$$

The work done by the ejected substance is

$$T_{e} = \frac{M_{e} \cdot V_{e}^2}{2}. \tag{65}$$

The efficiency of the rocket is

$$K_{p} = \frac{T}{T_{e}} = 1 : \left(1 + \frac{M_{e} \cdot V_{e}^2}{M_{i} \cdot V^2}\right), \tag{66}$$

and since $\frac{M_{e}}{M_{i}} = -\frac{V}{V_{e}}$, we have

$$K_{p} = 1 : \left(1 + \frac{V}{V_{e}}\right). \tag{67}$$

The last relation shows that the smaller the mass of the rocket relative to the ejected mass, the higher is the efficiency of the rocket. This equation was used to compute Table 13.

**TABLE 13.**

<table>
<thead>
<tr>
<th>Mass of rocket $M_{i}$</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejected mass $M_{e}$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Efficiency of rocket $K_{p}$</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Efficiency in %</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

Since the rocket always has a finite mass, the efficiency will never reach 100%.
However, if the rocket initially moves with a certain velocity, ejection of the propellant with the same velocity in the opposite direction will ensure complete utilization of its energy (100% efficiency). Indeed, let $V_i$ be the initial velocity of the rocket. Then

$$T = \frac{M_i(V_i + V_f)^2}{2} \quad ; \quad T_f = \frac{M_i(V_i + V_f)^2}{2},$$

(68)

and setting $\frac{M_1}{M_i} = -\frac{V}{V_i}$, we find

$$K_p = 1 + \frac{V_i(V + V_f)^2}{2(M(V + V_f)^2)},$$

(69)

so that for $V_f = -V_i$, this gives $K_p = 1$ or 100%.

It is therefore best to eject the propellant in the rearward direction against the motion of the rocket with the same velocity as the velocity of the rocket.

The above case is feasible if one of the following conditions is satisfied.

a) the energy is received from the earth in the form of radiation;
b) the energy is received from the sun in the form of alpha and beta particles;
c) the energy is supplied by a reserve of radium in the rocket.

In the last case, a very small mass of radium is needed because of the tremendous velocity of the particles ejected by this radioactive element, so that the mass of the rocket may be treated as constant.

The momentum equation for these cases of weightless energy sources is

$$M_i \cdot dV = V_i \cdot dM_i.$$

Integration gives

$$V = \frac{V_i}{M_i} M + C,$$

(70)

where $C$ is the initial velocity of the rocket.

If the initial velocity is zero, we have

$$V = \frac{M_1}{M_i} \cdot V_i.$$

(71)

If the velocity of ejection is $V_i = 3 \cdot 10^6$ m/sec and $M_1 = M_i$, we have $V = 3 \cdot 10^6$ m/sec, i.e., the velocity of the rocket is 18,000 times the escape velocity from the sun.

If $V_i = 30 \cdot 10^6$ and $V = 17 \cdot 10^6$, i.e., the velocity of the rocket is only slightly higher than the escape velocity from the sun, we have $\frac{M_f}{M_i} = 0.00057$, i.e., the mass of the radioactive element needed to propel the rocket is about 1/2,000 of the rocket mass.

The rocket efficiency is

$$K_p = 1 + \frac{M_1}{M_i}$$

(72)

or

$$\frac{M_1}{M_i + M_1} \approx \frac{M_1}{M_i}$$

(73)
In our case $K_e = 0.00057$, i.e., the utilization of energy is highly inefficient, but the quantity of the propellant being burnt is also negligible.

Case 2. The case of propellant driven by its intrinsic energy (e.g., fuels and explosives) is obviously more practicable. The energy reserve per unit ejected mass may be quite high in this case, ensuring a higher ejection velocity than with inert substances (e.g., sand) ejected by some extraneous explosive agent.

4. EXPLOSIVE ENERGY AND ITS UTILIZATION

To ensure optimum utilization of the explosive energy for rocket propulsion, the chemical energy of the particles should be efficiently converted into translational motion of gaseous or vaporous combustion products. The combustion products, having done useful work during their expansion inside the combustion ducts, should be ejected at a relatively low temperature; in this way, most of the thermal energy of the gases will have been used up in rocket propulsion. The absolute temperature of the exploding gases should preferably reach 10,000°, but in practice it can hardly exceed 3,000°. Table 14 lists the various efficiencies of utilization of thermal energy in the combustion ducts of a rocket. The numbers represent the potential energy, and not a measure of heat; above 2,000, the same numbers express the temperatures.

Table 14.

<table>
<thead>
<tr>
<th>Expansion of gases</th>
<th>1</th>
<th>6</th>
<th>36</th>
<th>216</th>
<th>1,300</th>
<th>7,800</th>
<th>46,800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute temperature or energy</td>
<td>10,000</td>
<td>5,000</td>
<td>2,500</td>
<td>1,250</td>
<td>625</td>
<td>312</td>
<td>156</td>
</tr>
<tr>
<td>Temperature (centigrade)</td>
<td>9,727</td>
<td>4,727</td>
<td>2,227</td>
<td>977</td>
<td>352</td>
<td>39</td>
<td>-147</td>
</tr>
<tr>
<td>Heat utilization in %</td>
<td>0</td>
<td>50</td>
<td>75</td>
<td>87</td>
<td>95</td>
<td>97</td>
<td>98.4</td>
</tr>
<tr>
<td>Loss in %</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>13</td>
<td>5</td>
<td>3</td>
<td>1.6</td>
</tr>
<tr>
<td>Approximate gas density relative to air</td>
<td>1,000</td>
<td>167</td>
<td>28</td>
<td>4.6</td>
<td>0.77</td>
<td>0.13</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Each successive column in the table corresponds to sixfold expansion of the gases compared to the preceding column, the absolute temperature dropping to one half. During flight in the atmosphere, the combustion should be conducted under high pressure, definitely not less than atmospheric. In vacuum, the pressure of the combustion products may be quite low. The gas pressure therefore should be regulated according to the actual flight conditions.

Calculating the cross section of a combustion chamber

Suppose the rocket is launched from a high mountain, where the atmospheric pressure is 0.3 kg/cm². The gases ejected from the nozzle should have this or higher pressure. Inside the combustion chamber, the gas
pressure obviously should be much higher, at least by a factor of 36 (assuming 75% utilization), i.e., 10 atmospheres, and in the lower air layers 10·3 = 30 atmospheres. We take 100 atm. If the rocket weighs 1 ton and the fuel brings its weight up to 5 tons, while the gas pressure is double the weight of the rocket, the gas pressure inside the combustion duct should be 10 tons, and the cross-sectional area is \( \frac{10 \times 1000}{100} = 100 \text{ cm}^2 \). The corresponding diameter is 11.3 cm.

Energy of explosive propellants

Table 15 lists the energy released by 1 kg of various explosives under two assumptions: a) 1 kg of propellant mixture contains the oxygen needed for combustion; b) the oxygen is taken from the surrounding air during flight through the atmosphere.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Large calories</th>
<th>kg-m</th>
<th>Velocity, m/sec</th>
<th>Work ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Without external supply of oxygen</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₂ and O₂: product, water vapor</td>
<td>3,200</td>
<td>1.37 ( \cdot 10^6 )</td>
<td>5,180</td>
<td>1.455</td>
</tr>
<tr>
<td>Same: product, water</td>
<td>3,736</td>
<td>1.6 ( \cdot 10^6 )</td>
<td>5,600</td>
<td>1.702</td>
</tr>
<tr>
<td>Same: product, ice</td>
<td>3,816</td>
<td>1.63 ( \cdot 10^6 )</td>
<td>5,650</td>
<td>1.730</td>
</tr>
<tr>
<td>C and O₂: product, CO₂</td>
<td>2,200</td>
<td>0.94 ( \cdot 10^6 )</td>
<td>4,290</td>
<td>1.006</td>
</tr>
<tr>
<td>Benzene C₆H₆ and O₂: product, H₂O and CO₂</td>
<td>2,370</td>
<td>1.01 ( \cdot 10^6 )</td>
<td>4,450</td>
<td>1.077</td>
</tr>
<tr>
<td><strong>B. With oxygen supplied from outside</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₂, product, H₂O</td>
<td>28,780</td>
<td>12.3 ( \cdot 10^6 )</td>
<td>15,520</td>
<td>13.08</td>
</tr>
<tr>
<td>C: product, CO₂</td>
<td>8,080</td>
<td>3.46 ( \cdot 10^6 )</td>
<td>8,240</td>
<td>3.673</td>
</tr>
<tr>
<td>Hydrocarbon</td>
<td>10,000</td>
<td>4.28 ( \cdot 10^6 )</td>
<td>9,160</td>
<td>4.545</td>
</tr>
<tr>
<td>Benzene; products, CO₂ and H₂O</td>
<td>13,000</td>
<td>5.56 ( \cdot 10^6 )</td>
<td>10,440</td>
<td>5,909</td>
</tr>
<tr>
<td>Radium</td>
<td>1.43 ( \cdot 10^9 )</td>
<td>0.611 ( \cdot 10^{12} )</td>
<td>3.44 ( \cdot 10^6 )</td>
<td>0.86 ( \cdot 10^6 )</td>
</tr>
</tbody>
</table>

5. SUPPLEMENT TO THE THEORY OF ROCKET MOTION IN VACUUM AND IN FREE SPACE

On p. 49 we derived the equation of rocket motion

\[
dV(M_1 + M) = - V_1 dM. \tag{74}
\]

which gives

\[
\frac{dV}{dt} = - \frac{V_1}{M_1 + M} \cdot \frac{dM}{dt}. \tag{75}
\]
The first part is the acceleration of the rocket. It is proportional to the amount of fuel burnt in unit time. Moreover, the acceleration increases as the remaining quantity of fuel $M$ decreases.

To achieve constant acceleration, corresponding to constant relative gravitation $T_0$ inside the rocket, we have to satisfy the condition

$$\frac{V_i}{M_i + M} \cdot \frac{dM}{dt} = T_0.$$  \hfill (76)

whence

$$\frac{V_i}{M_i + M} \cdot \frac{dM}{dt} = T_0 \cdot dt.$$  \hfill (77)

Integration gives

$$V_i \cdot \ln \left( \frac{M_i + M}{M_i} \right) = T_0 \cdot t + C \ldots \ldots \text{for } t_0 \text{ and } C = 0.$$  \hfill (78)

When the entire fuel reserve has burnt out, $M = 0$ and

$$t = \frac{V_i}{T_0} \ln \left( 1 + \frac{M}{M_i} \right),$$  \hfill (79)

i.e., the fuel burning time is inversely proportional to the relative gravitation and increases with the fuel mass $M$.

We have from (76)

$$\frac{dM}{dt} = \frac{M_i + M}{V_i} T_0 \cdot \frac{1}{M_i}.$$  \hfill (80)

It follows from this equation that the fuel expenditure will be minimum at the end of the flight, when $M$ is small, and large at the beginning of the flight, when $M$ is large. At the end of the flight,

$$\frac{dM}{dt} = \frac{M_i}{V_i} T_0,$$  \hfill (81)

and at the beginning

$$\frac{dM}{dt} + \frac{M_i + M}{V_i} T_0,$$  \hfill (82)

so that the ratio of the two rates is

$$\frac{M_i + M}{M_i} = 1 + \frac{M}{M_i}.$$  \hfill (83)

The higher the ratio $\frac{M}{M_i}$, the more pronounced is the change in fuel burning rate; conversely, the burning rate is almost constant when this ratio is small. In practice, the propulsion forces are difficult to vary; it is simpler to reduce the relative gravitation, by immersing the crew and other delicate objects in liquid baths.

- Tsiolkovskii's work contains an error, giving $\frac{dM}{dt} = \frac{M_i + M}{V_i + T_0}$ (equation (3g1), p.39), but this does not affect the subsequent computations.
If the fuel burning rate is constant, the total burning time is

$$t = M_1 \frac{dM}{dt}$$  \hspace{1cm} (84)$$

where $M_1$ is the total fuel reserve.

Here the derivative $\frac{dM}{dt}$ is the constant fuel burning rate (per second).

If the fuel burning rate is variable, while the relative gravitation is kept constant, the total burning time will be

$$t = \frac{V}{\text{acceleration}} = V \frac{dV}{dt}$$  \hspace{1cm} (85)$$

where $\frac{dV}{dt}$ is constant.

Table 16 below is an expanded version of Table 2 (p. 52) and is based on equations (6) and (7).

<table>
<thead>
<tr>
<th>Ratio of fuel mass to rocket mass $\frac{M_f}{M_r}$</th>
<th>Velocity $V$, m/sec, if $V_f = 5,000$ m/sec (eq. (6))</th>
<th>Velocity $V$, m/sec if velocity $V_f = 4,000$ m/sec (eq. (5))</th>
<th>Utilisation $K_p$ in % (eq. (7))</th>
<th>Altitude in km for constant terrestrial gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>472.5</td>
<td>378</td>
<td>8.87</td>
<td>11.4</td>
</tr>
<tr>
<td>0.2</td>
<td>940</td>
<td>728</td>
<td>16.55</td>
<td>42</td>
</tr>
<tr>
<td>0.3</td>
<td>1,810</td>
<td>1,048</td>
<td>22.9</td>
<td>92</td>
</tr>
<tr>
<td>0.4</td>
<td>1,680</td>
<td>1,344</td>
<td>28.2</td>
<td>138</td>
</tr>
<tr>
<td>0.5</td>
<td>2,025</td>
<td>1,620</td>
<td>32.8</td>
<td>204</td>
</tr>
<tr>
<td>0.6</td>
<td>2,345</td>
<td>1,876</td>
<td>36.7</td>
<td>280</td>
</tr>
<tr>
<td>0.7</td>
<td>2,645</td>
<td>2,116</td>
<td>40.0</td>
<td>357</td>
</tr>
<tr>
<td>0.8</td>
<td>2,990</td>
<td>2,344</td>
<td>42.9</td>
<td>440</td>
</tr>
<tr>
<td>0.9</td>
<td>3,210</td>
<td>2,568</td>
<td>45.8</td>
<td>520</td>
</tr>
<tr>
<td>1.0</td>
<td>3,465</td>
<td>2,772</td>
<td>48.0</td>
<td>607</td>
</tr>
<tr>
<td>1.5</td>
<td>4,575</td>
<td>3,060</td>
<td>55.8</td>
<td>650</td>
</tr>
<tr>
<td>2.0</td>
<td>5,490</td>
<td>4,392</td>
<td>60.3</td>
<td>1,520</td>
</tr>
<tr>
<td>3</td>
<td>6,900</td>
<td>5,520</td>
<td>63.5</td>
<td>2,430</td>
</tr>
<tr>
<td>4</td>
<td>8,045</td>
<td>6,436</td>
<td>64.7</td>
<td>3,300</td>
</tr>
<tr>
<td>5</td>
<td>8,960</td>
<td>7,168</td>
<td>64.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>9,730</td>
<td>7,784</td>
<td>63.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10,385</td>
<td>8,316</td>
<td>61.7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10,985</td>
<td>8,788</td>
<td>60.5</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>11,615</td>
<td>9,212</td>
<td>58.9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>11,990</td>
<td>9,592</td>
<td>57.6</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>13,865</td>
<td>11,092</td>
<td>51.2</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>15,220</td>
<td>12,176</td>
<td>46.3</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>17,170</td>
<td>13,736</td>
<td>39.3</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>22,400</td>
<td>17,920</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>26,281</td>
<td>21,040</td>
<td>21.0</td>
<td></td>
</tr>
<tr>
<td>193</td>
<td>30,038</td>
<td>24,032</td>
<td>14.4</td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The actual altitude is greater, since the gravitation decreases.
A velocity of 5,000 m/sec corresponds to a hydrogen-oxygen mixture, whereas a velocity of 4,000 m/sec corresponds to carbon fuel combining endogenously with oxygen.

The above analysis was carried out for free-space conditions without gravitation (between stars or galaxies), for conditions of low gravitation (on small asteroids), and for rocket flight outside the atmosphere not touching on other planets or entering their atmospheres (in the latter case, all the phenomena are relative).

6. INCLINED FLIGHT
(Supplement to p. 59)

Tsiolkovskii writes equation (29) (p. 59) in the form

$$ R = V^2 p^2 + g^2 - 2 pg \sin (a + \beta - 90^\circ). $$  \hspace{1cm} (86)

The work of the rocket is

$$ T_i = \frac{M_1 \cdot V_1^2}{2} + T_1. $$  \hspace{1cm} (87)

where

$$ T_1 = g \cdot M_1 \cdot L \cdot \sin (a - 90^\circ) $$  \hspace{1cm} (88)

is the work done to lift the rocket.

If $R$ and $P$ are constant, the distance $L$ is

$$ L = \frac{V_1^2}{2R}. $$  \hspace{1cm} (89)

From (87)–(89) we have

$$ T_i = \frac{1}{2} \cdot M_1 \cdot V_1^2 \cdot \{1 + \sin (a - 90^\circ) \frac{R}{p}\}. $$  \hspace{1cm} (90)

The work of the propellant is

$$ T_2 = \frac{M_2 \cdot V_i^2}{2}. $$  \hspace{1cm} (91)

From (90) and (91) we have for the rocket efficiency

$$ K_i = \frac{T_i}{T_2} = \frac{M_1}{M_2} \cdot \frac{V_1^2}{V_i^2} \cdot \{1 + \sin (a - 90^\circ) \frac{R}{p}\}. $$  \hspace{1cm} (92)

Figure 33 shows that

$$ \cos a = -\sin (a - 90^\circ) = \frac{g - p \sin (a + \beta)}{V^2 p \cdot \cos (a + \beta - 90^\circ) + \{g - \sin (a + \beta - 90^\circ)\}^2}. $$  \hspace{1cm} (93)

The unknown $\sin (a - 90^\circ)$ can now be eliminated from (92).
To eliminate $V_z$, we use the relation

$$t = \frac{V_z}{V_0} \ln \left(1 + \frac{M_3}{M_1}\right). \tag{94}$$

This relation gives the total fuel burning time for constant relative gravitation $T_s$. But $T_s = P$ and $V_z = R \cdot t$, so that we have from (94)

$$V_z^2 = \frac{R^2}{P^2} \cdot \frac{V_0^2}{V_0} \ln \left(1 + \frac{M_3}{M_1}\right)^2. \tag{95}$$

From (92), (93), and (95) we have

$$K_p = \frac{R^2}{P^2} \cdot \frac{M_1}{M_1} \ln \left(1 + \frac{M_3}{M_1}\right)^2 \cdot \left\{1 + \frac{g \sin (\alpha + \beta - 90^\circ)}{\sqrt{P^2 \cos^2 (\alpha + \beta - 90^\circ) + [g - P \sin (\alpha + \beta - 90^\circ)]^2}} \cdot \frac{\sqrt{P^2 + g^2 - 2 \cdot g \sin (\alpha + \beta - 90^\circ)}}{g}\right\}. \tag{96}$$

If $g = 0$ and $R = P$, we obtain equation (7).

For vertical flight, $\alpha + \beta - 90^\circ = 90^\circ$ and $R = P - g$. The efficiency of the rocket is then

$$K_p = \frac{M_1}{M_1} \ln \left(1 + \frac{M_3}{M_1}\right)^2 \left(1 - \frac{g}{P}\right). \tag{97}$$

Using (18), Tsiolkovskii obtained the efficiencies $K_p = \frac{T_1}{T_0}$ for various values of $p / g$. These results, together with the corresponding speeds (in %) are listed in Table 17.

<table>
<thead>
<tr>
<th>$p/g$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency</td>
<td>0</td>
<td>50</td>
<td>66.7</td>
<td>75</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>Speed, %</td>
<td>0</td>
<td>70.7</td>
<td>81.7</td>
<td>86.0</td>
<td>89.4</td>
<td>94.9</td>
<td>100</td>
</tr>
</tbody>
</table>

7. FLIGHT IN A GRAVITATIONAL FIELD AND IN AIR
(Approximate treatment)

If the flight velocity is horizontal or makes an angle not exceeding $40^\circ$ with the horizon, the air drag may be treated as a downward force which appears to make the rocket fall. The corresponding fall velocities, however, range from 20–30 m/sec to 1 m/sec and less. This is clearly quite negligible compared with the tremendous velocities of the rocket.

84
We have from Figure 36

\[ V_z = R \cdot t \]  
\[ R = p - g \cdot \sin \beta \]  
\[ T_0 = p \]  
\[ t = \frac{V_z}{T_0} \ln \left(1 + \frac{M_i}{M} \right) \]  
\[ V_z = (p - g \sin \beta) \cdot \frac{V_z}{T_0} \ln \left(1 + \frac{M_i}{M} \right). \]

Here \( p \) is constant.

Let us find the efficiency of the rocket:

\[ T = \frac{M_1 \cdot V_z^2}{2} + T_3 \]  
\[ T_3 = M_1 \cdot g \cdot h = M_1 \cdot g \cdot L \sin \beta. \]

92 Hence

\[ T = \frac{M_1 \cdot V_z^2}{2} \left\{1 + \frac{g}{R} \sin \beta \right\}. \]

Moreover,

\[ T_3 = \frac{M_1^2}{2} \cdot V_z^2. \]

The rocket efficiency is thus

\[ K_p = \frac{T}{T_3} = \frac{M_1}{M} \cdot \frac{V_z^2}{V_z^2} \left\{1 + \frac{g}{R} \sin \beta \right\}. \]

Equations (99), (101), and (106) yield

\[ K_p = \frac{M_1}{M_2} \ln \left(1 + \frac{M_i}{M} \right)^2 \cdot \left\{1 - \frac{g}{R} \sin \beta \right\}. \]

The loss relative to free space is

\[ \frac{g}{R} \sin \beta. \]

If we take \( \frac{p}{\dot{q}} = 0.3; \ \beta = 20^\circ; \ \sin \beta = 0.342 \), the loss will reach 11.4%.

Table 18 lists the losses for various flight angles.

<table>
<thead>
<tr>
<th>Flight angle, deg</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy losses in %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.17</td>
<td>0.34</td>
<td>0.85</td>
<td>1.7</td>
<td>2.6</td>
<td>3.4</td>
<td>4.2</td>
<td>5</td>
<td>5.7</td>
</tr>
<tr>
<td>5</td>
<td>0.34</td>
<td>0.64</td>
<td>1.7</td>
<td>3.4</td>
<td>5.2</td>
<td>6.8</td>
<td>8.4</td>
<td>10</td>
<td>11.4</td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>1.7</td>
<td>4.25</td>
<td>8.5</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>25</td>
<td>28.5</td>
</tr>
<tr>
<td>1</td>
<td>1.7</td>
<td>3.4</td>
<td>8.5</td>
<td>17</td>
<td>26</td>
<td>34</td>
<td>42</td>
<td>50</td>
<td>57</td>
</tr>
</tbody>
</table>
8. MORE EXACT COMPUTATION OF AIR DRAG

Suppose that the air temperature is constant and the atmosphere is infinite. Then

\[ h = \frac{f}{d_i} \ln \left( \frac{d_i}{d} \right). \]  

(109)

where \( \frac{f}{d_i} \) is the height \( h \), of the imaginary atmosphere with constant density. Therefore

\[ \frac{h}{h_i} \ln \left( \frac{d_i}{d} \right) \]  

(110)

and

\[ d = d_i \cdot e^{-\frac{h}{h_i}}. \]  

(111)

This relation is a simplified form of equation (46). The air drag is

\[ F_i = \frac{s \cdot d \cdot v^2}{U \cdot 2 \cdot R} \text{ ton}. \]  

(112)

In oblique motion, the path length is (Figure 36)

\[ L = h \cdot \sin \beta. \]  

(113)

By (99)

\[ R = p - g \cdot \sin \beta \]

and also

\[ V = \sqrt{2 \cdot R \cdot h}. \]  

(114)

Therefore

\[ V = \sqrt{2 \left( p - g \cdot \sin \beta \right) L}. \]  

The work element of air drag is

\[ dT_i = F_i \cdot d \cdot L. \]  

(115)

This relation, combined with equations (111)-(114), gives

\[ dT_i = \frac{s \cdot d_i}{U \cdot k} \left( p - \sin \beta \right) \cdot L \cdot e^{-\frac{L \cdot \sin \beta}{h_i}} \cdot d \cdot L. \]  

(116)

Let

\[ \frac{L \cdot \sin \beta}{h_i} = h = x \text{ (see (113))}; \]

\[ dx = \frac{\sin \beta}{h_i} \cdot dL = \frac{dh_i}{dh_i} \cdot dL = \frac{h_i}{\sin \beta} \cdot dx. \]

Then

\[ dT_i = \frac{s \left( p - g \cdot \sin \beta \right) \cdot d_i}{U \cdot k \cdot \sin \beta} \cdot h_i^2 \cdot e^{-x} \cdot dx. \]  

(117)
We write
\[ \frac{s (p - g \sin \beta)}{U \cdot g \sin^2 \beta} \, d_1 \, h_1, = A. \] (118)

Integrating, we obtain (see (113))
\[ T_i = A \left\{ 1 - \left(1 + \frac{h}{h_i \sin \beta} \right) e^{\frac{-h}{h_i \sin \beta}} \right\} = A \left\{ 1 - \left(1 + \frac{L}{h_i \sin \beta} \right) e^{\frac{-L}{h_i \sin \beta}} \right\}. \] (119)

Let
\[ \frac{L}{h_i \sin \beta} = \frac{h}{h_i \sin \beta} = Z. \] (120)

Then
\[ T_i = A \left\{ 1 - \left(1 + z \right) e^{-z} \right\}. \] (121)

For total air-drag work \( B = \infty \) or \( z = +\infty \). We have
\[ e^{-z} = 1; e^{z} = 1; \left(1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \right), \] (122)

so that
\[ \left(1 + z \right) e^{-z} = e^{-z} + z \cdot e^{-z} = e^{-z} + z \left(1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \right) = \]
\[ = \frac{1}{e^z} + 1; \left(\frac{1}{z} + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \right). \] (123)

94 If \( B \) or \( z \) are infinity, expression (123) vanishes.
The air-drag work is thus
\[ T_i = A. \] (124)

The total work of vertical motion is derived from (118) by setting \( \beta = 90^\circ \).
This gives
\[ T_s = \frac{S \cdot (p - g)}{U \cdot g} \cdot d, h_1. \] (125)

Comparing this work with the total work of oblique motion, we find that the ratio of the latter to the former is
\[ \frac{p - g \sin \beta}{(p - g) \sin^2 \beta} \cdot \] (126)

For small angle \( \beta \), the above relations are inapplicable.
Equation (118) can be applied to compute the total work of vertical motion. Thus, for
\[ S = 2 \, m^3; \, p = 100; \, g = 10; \, h_1 = 8,000; \, d_i = 0.0013; \, U = 100, \]

87
we have

\[ T_i = 14,976 \text{ ton-m.} \]

This work is small compared with the work of the rocket, which exceeds 60 million ton-m. In oblique motion it increases. Equation (126) with \( p = 30 \) or 20 and \( g = 10 \) was used to compute the figures listed in Table 19 for various \( \beta \).

**TABLE 19.**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i: p = 30 )</td>
<td>46.7</td>
<td>11.3</td>
<td>5.0</td>
<td>2.85</td>
<td>1.92</td>
<td>1.0</td>
</tr>
<tr>
<td>( T_i: p = 20 )</td>
<td>60</td>
<td>14.2</td>
<td>6.0</td>
<td>3.3</td>
<td>2.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( 1 : \sin^2 \beta )</td>
<td>33</td>
<td>8.55</td>
<td>4.0</td>
<td>2.42</td>
<td>1.00</td>
<td>1.0</td>
</tr>
</tbody>
</table>

We see from the second line that a 20° inclination increases the work by a factor of 11; comparison of the second and third lines with the fourth line shows that the work is roughly proportional to \( 1/\sin^2 \beta = \csc^2 \beta \).

The dependence of the air-drag work on the path length or the final altitude \( h \) is listed in Table 20 (from equations (118), (121), and (122)).

**TABLE 20.**

<table>
<thead>
<tr>
<th>Angle ( \beta )</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h = 4 ) km</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3.6</td>
<td>25</td>
<td>45</td>
<td>59</td>
<td>74</td>
</tr>
<tr>
<td>( h = 8 ) km</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.094</td>
<td>54</td>
<td>19</td>
<td>32</td>
<td>55</td>
</tr>
<tr>
<td>( h = 16 ) km</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
<td>24</td>
<td>79</td>
<td>25</td>
</tr>
<tr>
<td>( h = 24 ) km</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.305</td>
<td>2</td>
<td>11</td>
</tr>
</tbody>
</table>

95. **OPTIMAL FLIGHT ANGLE**

Equations (98) or (108) may be applied to compute the loss of work in oblique flight in a gravitational field. Equation (118) gives the corresponding losses due to air drag. We can now determine the optimal flight angle.

The loss due to the oblique motion of the rocket is given by (108):

\[ \frac{E}{p} \cdot \sin \beta \]  (in absol. units). \hspace{1cm} (127)

The air drag losses in absolute units are obtained from (118):

\[ A \cdot g = \frac{1(p - g \sin \beta)}{U \cdot \sin^2 \beta} \cdot d_i \cdot h_i^2 \]  (128)
The work of the rocket is

\[ T_1 = \frac{M_i}{2} \cdot \frac{V_i^2}{\ln \left(1 + \frac{M_i}{M_f}\right)^2}. \]  

(129)

The two loss components in absolute units are therefore

\[ T_i \cdot \frac{g}{p} \sin \beta + A \cdot g = T_i \cdot \frac{g}{p} \sin \beta + \frac{s}{U} \cdot d_i \cdot h_i \cdot \left[ \frac{p - g \sin \beta}{\sin^2 \beta} \right] = z. \]  

(130)

Differentiating this equation and setting the derivative equal to zero, we obtain a complicated equation which is difficult to solve for \( \sin \beta \).

We saw on p. 68 that the optimal angle is fairly small. We can therefore drop the term \( g \sin \beta \) in braces, so that equation (130) takes the form

\[ z = T_i \cdot \frac{g}{p} x + \frac{s}{U} \cdot d_i \cdot h_i \cdot \frac{p}{x}. \]  

(131)

Here \( x = \sin \beta \).

Differentiating and equating the first derivative to zero, we find

\[ x = \sin \beta = \sqrt[3]{\frac{4 \cdot s \cdot d_i \cdot h_i}{U \cdot T_i} \cdot \frac{p^3}{g}}. \]  

(132)

and using (129)

\[ \sin \beta = \sqrt[3]{\frac{4 \cdot s \cdot d_i \cdot h_i}{U \cdot T_i} \cdot \frac{p^3}{g} \cdot \ln \left(1 + \frac{M_i}{M_f}\right)^2 \cdot g}. \]  

(133)

We see that the optimal angle \( \beta \) increases with the propulsion energy \( p \) and with the frontal area of the rocket \( s \) and decreases as the shape coefficient \( U \) and the mass ratio \( \frac{M_i}{M_f} \) increase.

For example, for \( s = 2; \ d_i = 0.0013; \ h_i = 8,000; \ \frac{p}{x} = 10; \ U = 100; \ T_i = 10; \ V_i = 5,000 \), we find \( \sin \beta = 0.167 \) and \( \beta = 9^\circ 35' \). For \( p = 20, \sin \beta = 0.57 \) and \( \beta = 3^\circ 20' \).

But for such small angles the air drag will be much less (because of the spherical shape of the atmosphere), so that the true optimal angle is even smaller.

From (131) we obtain the relative loss due to the two factors:

\[ \frac{z}{T_i} = \frac{\frac{g}{p} x + \frac{s}{U} \cdot d_i \cdot \frac{p}{x} h_i^2}{U \cdot T_i \cdot V_i^2 \cdot \ln \left(1 + \frac{M_i}{M_f}\right)^2 \cdot x^2}. \]  

(134)

To simplify this expression, we divide the second term by the third term. We thus obtain the ratio of gravitational losses to air-drag losses. Then we use (133) to eliminate \( x \) from this expression. The result is 2, so that for the optimal flight angle, the gravitational losses are double the air-drag losses. Thus,

\[ z : T_i = \frac{\frac{g}{p} x + \frac{s}{2p} x = \frac{3}{2} \cdot \frac{g}{p} V.} { \frac{\frac{g}{p} x + \frac{s}{2p} x = \frac{3}{2} \cdot \frac{g}{p} V.} \]  

(135)
For flight angles of 9° and 3°, we obtain total losses of 0.025 and 0.0428, i.e., 2.5% and 4.3%, respectively.

From (135) and (133) we find the total relative loss:

$$z \cdot T_1 = \sqrt[3]{\frac{2T \cdot 3 \cdot d_i \cdot h_i^2 \cdot e^\gamma}{2 \cdot U \cdot T \cdot V_i \ln \left(1 + \frac{h_i}{H_i}\right)^2 \cdot \rho}}$$

(136)

Table 21 lists the percentage losses for various $p$ and $x$.

| TABLE 21. |
|-----------------|---|---|---|---|---|---|---|---|---|---|
| Free-space acceleration, $(p)$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| sin $\beta = x$ ... | 0.0097 | 0.0154 | 0.0204 | 0.0246 | 0.0292 | 0.0326 | 0.0356 | 0.0392 | 0.0422 | 0.0453 |
| Angle $\beta$, deg ... | 0°56' | 0°88' | 1'17' | 1'41' | 1'68' | 1'86' | 2°07' | 2°26' | 2°43' | 3°00' |
| Percentage losses $z \cdot T_1$ ... | 14.6 | 11.6 | 10.2 | 9.23 | 8.57 | 8.07 | 7.66 | 7.33 | 7.05 | 6.80 |

Free-space acceleration, $(p)$

| sin $\beta = x$ ... | 0.059 | 0.072 | 0.083 | 0.094 | 0.114 | 0.133 | 0.150 | 0.162 | 0.211 | 0.333 |
| Angle $\beta$, deg ... | 3°25' | 4°10' | 4°45' | 5°25' | 6°33' | 7°40' | 8°40' | 10°30' | 12°10' | 19°30' |
| Percentage losses $z \cdot T_1$ ... | 5.94 | 5.40 | 4.97 | 4.71 | 4.28 | 3.96 | 3.75 | 3.40 | 3.10 | 2.50 |

The numerical data of the table, used in conjunction with (128), readily show that the approximate relations are fairly accurate even for $p = 1$. For large $p$, the error is negligible.

97 10. THE EFFECT OF EARTH CURVATURE

From (111), (112), (114), and (115), we have in the usual units

$$dT_e = \frac{d}{U \cdot e^{-\frac{h}{k}}} \left(p - \sin \beta \cdot g\right) \cdot e^{-\frac{h}{k}} \cdot L.$$  

(137)

For a flat earth, we also had equation (113), $L = h / \sin \beta$. For a spherical earth, however, this relation applies to very small angles $\beta$ only. A more exact relation, applicable to any angle, is

$$h = L \sin \beta + \frac{L^2}{2r} = L \left(\sin \beta + \frac{L}{2r}\right),$$  

(138)

where $r$ is the earth's radius.
Hence,

\[ L = -r_2 \sin \beta \left(1 - \sqrt{1 + \frac{2h}{r_2 \sin \beta}}\right). \]  \hfill (139)

We take

\[ \frac{2h}{r_2 \sin \beta} = x; \sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^4}{16}. \]  \hfill (140)

Retaining the first three terms only, we obtain

\[ L = -r_2 \sin \beta \left(1 - \frac{x}{2} + \frac{x^2}{8}\right) = \frac{h}{\sin \beta} - \frac{4h^2}{r_2 \sin^2 \beta} = \frac{h}{\sin \beta} \left(1 - \frac{h}{2r_2 \sin^2 \beta}\right). \]  \hfill (141)

We will solve the problem of air-drag work for the particular case of horizontal flight ($\beta = 0$). Then

\[ h = \frac{r_2^2}{2r_2} L = V^2, \quad r_2 \cdot h. \]  \hfill (142)

From (116) we thus have

\[ dT = \frac{s}{U \cdot g} \cdot \frac{e^{-h}}{h_1} \cdot L dL = \frac{s}{U \cdot g} \cdot \frac{e^{-\frac{L^2}{2r_2^2}}}{h_1} \cdot L dL \]  \hfill (see 142)

(143)

Let

\[ \frac{L^2}{2r_2^2} = x. \]

Then

\[ L \cdot dL = r_2 h_1 \cdot dx \]  \hfill (144)

and (143) is replaced with

\[ dT = \frac{s}{U \cdot g} \cdot \frac{d_1}{h_1} \cdot r_2 \cdot h_1 \cdot e^{-x} dx = A \cdot e^{-x} dx. \]  \hfill (145)

Integration gives

\[ T = A \left(1 - e^{-x}\right) = A \left(1 - e^{-h_1}\right) = A \left(1 - 2e^{-\frac{L^2}{2r_2^2}}\right); \]  \hfill (146)

\[ A = \frac{s}{U \cdot g} \cdot d_1 \cdot r_2 \cdot h_1. \]  \hfill (147)

This expression gives the total work of air drag.

In vertical flight, we had

\[ T = \frac{s(p \cdot g)}{U \cdot g} \cdot d_1 \cdot h_1. \]  \hfill (125)
The corresponding air-drag work is less (by (125) and (147)):

$$\frac{p}{p-g} \cdot \frac{r_s}{h_s} \text{ times.}$$ \hspace{1cm} (148)

For $p = 100$, $g = 10$, and $h_s = 8,000$, we find 883 from (148), i.e., the air-drag work in horizontal motion is almost a factor of 1,000 higher than the corresponding air-drag work for vertical flight. A nearly horizontal trajectory is thus highly unfavorable. We see from (146) and (147) that $T_s$ is highly sensitive to $p$. As an example, let us compute the air-drag work for horizontal flight assuming various $p$ and taking $s = 2$, $U = 50$ (equation (146)):

$$T_s = 264,800p.$$ \hspace{1cm} (149)

The work of the rocket is

$$T_i = \frac{M_i}{2} \cdot V_i^2 \ln \left[1 + \frac{M_i}{M_r}\right].$$ \hspace{1cm} (150)

<table>
<thead>
<tr>
<th>Table 22. Propulsion force $p$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage losses</td>
<td>0.42</td>
<td>0.83</td>
<td>2.1</td>
<td>4.2</td>
<td>8.3</td>
<td>12.5</td>
<td>20.3</td>
<td>41.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 23. Air drag and rocket work for various constant flight velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>The table is based on the following assumptions: rocket mass 10 ton, frontal area $4 \text{ m}^2$, shape coefficient 0.01, air density $0.0013$ of water density</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Air drag on frontal area, in tons, $R_m = 0.0001 V^2 \cdot \ldots$</td>
</tr>
<tr>
<td>Air drag for the motion of the entire rocket $R = 0.01 R_m$, $\ldots$</td>
</tr>
<tr>
<td>Work of rocket for 10 km range, thousands ton-m</td>
</tr>
</tbody>
</table>

If the rocket weighs 10 t, the work needed to escape the earth's gravitational pull is at least $6,370,000 \times 10 \times 2 = 127,400,000$, where the factor 2 indicates that at most 50% of the propellant energy is effectively utilized.

<table>
<thead>
<tr>
<th>(99) Ratio of air-drag work to propellant work over 10 km distance, %</th>
<th>10</th>
<th>113</th>
<th>2.02</th>
<th>3.15</th>
<th>4.54</th>
<th>8.06</th>
<th>9.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same, relative to the kinetic energy of the rocket, %</td>
<td>1.00</td>
<td>2.26</td>
<td>4.04</td>
<td>6.30</td>
<td>9.08</td>
<td>16.12</td>
<td>18.20</td>
</tr>
<tr>
<td>Same, for a 50 km range, %</td>
<td>5.00</td>
<td>11.50</td>
<td>20.2</td>
<td>31.5</td>
<td>45.4</td>
<td>80.6</td>
<td>91.0</td>
</tr>
<tr>
<td>Same, for an empty rocket of 1 ton, %</td>
<td>50</td>
<td>113</td>
<td>202</td>
<td>315</td>
<td>454</td>
<td>806</td>
<td>910</td>
</tr>
<tr>
<td>Same, for a 10 ton projectile fired at an altitude of 8 km to a range of 50 km, %</td>
<td>1.5</td>
<td>3.4</td>
<td>6.0</td>
<td>9.4</td>
<td>13.6</td>
<td>24.2</td>
<td>27.3</td>
</tr>
</tbody>
</table>

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The work needed to escape from the earth's gravitational pull for \( M_i = 10 \) is about \( 64 \cdot 10^6 \). This exceeds the air drag by a factor of \( \frac{240}{P} \).

The computed losses are listed in Table 22.

99 11. LANDING ON THE EARTH AND THE PLANETS

If a rocket of mass \( M_i \) requires \( K_i \) times its mass of fuel in order to be lifted from the ground, the total mass is

\[
M_i (1 + K_i).
\]

(151)

Soft landing will require the additional quantity of fuel

\[
M_i (1 + K_i) K_i.
\]

so that the total launch mass will be

\[
M_i (1 + K_i)^2,
\]

and the fuel mass is

\[
M_i (1 + K_i)^3 - M_i = M_i [(1 + K_i)^3 - 1].
\]

(152)

If

\[ M_i = 1 \] and \[ K_i = 9, \]

the fuel reserve will be 99 times the mass of the rocket with all its contents, except the fuel. Such a reserve is hardly a practicable proposition.

The quantity of fuel needed to launch a rocket from the earth and to land it on another (hypothetical) planet orbiting at the same distance from the sun is

\[
M_i [(1 + K_i) (1 + K_i) - 1],
\]

(153)

where \( \lambda_i \) is the relative quantity of the propellant needed for the landing.

If it is impossible to refuel on the target planet and the rocket should return to earth, the initial fuel reserve should be

\[
M_i [(1 + K_i)^3 (1 + K_i)^3 - 1]
\]

(154)

If the mass and the volume of the counterpart planet are equal to those of the earth, we have

\[
M_i [(1 + K_i)^3 - 1].
\]

(155)
Finally, taking
\[ M_1 = 1 \text{ and } K_1 = 9, \]
we obtain an initial fuel reserve of 9,999 \( M_1 \), which is clearly unfeasible.

Trips to Venus, Jupiter, and other major planets are therefore unfeasible; conversely, a flight to the asteroids is much easier to accomplish.

A round trip to \( n \) different planets (including the earth) without refuelling on landing, requires the total fuel reserve

\[ M_1 \left(1 + K_1\right) \left(1 + K_2\right)^2 \left(1 + K_3\right)^3 \ldots \left(1 + K_n\right)^n - 1 \]. \tag{156} \]

This tremendous fuel reserve can be somewhat reduced by settling for unpowered landings, with the rocket coasting through the air of each planet as an aircraft.

12. HORIZONTAL FLIGHT WITH AN INCLINED LONGITUDINAL AXIS OF THE ROCKET IN AN ATMOSPHERE WITH CONSTANT AIR DENSITY

Let the rocket fly horizontally with a velocity \( V \), so that its longitudinal axis makes an angle \( \beta \) with the horizon. The vertical air pressure on the rocket is then (Figure 37):

\[ R_y = \frac{d}{S} \cdot S' \cdot K_c \cdot \sin \beta \cdot V', \tag{157} \]

where \( S' \) is the horizontal projection of the rocket, \( K_c \) is a correction coefficient, comparing it with a non-prolate plane.

In equilibrium, the weight of the rocket \( M_i \) is equal to the vertical air pressure.

Therefore, from (157)

\[ \sin \beta = \frac{M_i \cdot d}{S' \cdot K_c \cdot V'}. \tag{158} \]

If \( M_i = 1; \ g = 10; \ d = 0.0013; \ V = 100; \ S' = 20; \ K_c = 1, \) we have \( \beta = 2.2^\circ \).

For \( M_i = 10; \ \beta = 22.7^\circ \).
For \( V = 1,000; \ \beta' = 0.001 \beta \).

Let us now find the work \( T_i \) of the horizontal air drag \( R_y \). We have

\[ R_y = R_y \cdot \sin \beta = M_i \cdot \sin \beta = \frac{M_i \cdot \epsilon}{d \cdot S' \cdot K_c \cdot V'}. \tag{159} \]

The work element is

\[ dT_i = R_y \cdot dL. \tag{160} \]
Taking \( d \) constant and \( p \) variable, we obtain

\[
V = \frac{V_0}{2p} \cdot L.
\]  

(161)

From (159)–(161), we have

\[
dT = \frac{M_i' \cdot \frac{g}{2} \cdot dL}{d \cdot S_i \cdot K_e \cdot p \cdot L}.
\]  

(162)

Integration gives

\[
T_s = A \cdot \ln \left[ \frac{L}{L_i} \right].
\]  

(163)

Here

\[
A = \frac{M_i' \cdot \frac{g}{2} \cdot S_i \cdot K_e \cdot p}{L}.
\]  

(164)

The work reckoned from the beginning of the trajectory (where the velocity is zero) is infinite. The work, however, becomes relatively small once the rocket has covered part of its trajectory \( L_i \), and has acquired a certain velocity. In a constant-density medium the work increases to infinity.

In equation (164) we take \( M_i' = 1; \ g = 10; \ S_i = 20; \ K_e = 1; \ p = 10 \). Then \( A = 19.2 \) and

\[
T_s = 19.2 \ln \left[ \frac{L}{L_i} \right].
\]  

(165)

Suppose the rocket covers 1,000 km after the initial leg of 10 km. Then \( T_s = 19.2 \ln 100 = 88.3 \). If the first leg is 1 km, we have \( T_s = 132.5 \). Thus a truly negligible work is needed to prevent the rocket from falling. We will now express this work as a function of \( V \).

From (161) and (163) we have

\[
L = \frac{V_i}{2p} \quad \text{and} \quad T_s = A \ln \left[ \frac{V_f}{V_i} \right].
\]  

(166)

If the initial velocity is \( V_i = 100 \text{ m/sec} \) and the final velocity is \( V = 10,000 \text{ m/sec} \), we have

\[
T_s = 19.2 \ln (100^6) = 176.6.
\]

If the initial velocity is \( V_i = 10 \text{ m/sec} \), we have

\[
T_s = 19.2 \ln (1,000^6) = 265.
\]

The path length is obtained from (161):

\[
L = \frac{V_i}{2p} = 5.10^4 \text{ m} = 5,000 \text{ km}.
\]

The work clearly depends on air density: it vanishes in vacuum and reaches tremendous values in the lower dense layers of the atmosphere. In general, if the rocket moves through a constant-density atmosphere, the
centrifugal forces may balance the gravitational and the angle may be taken equal to zero. The air drag will increase in this case, however. Moreover, we have no intention of trying to prove that the trajectory through constant-density air is optimal.

13. FLIGHT WITH THE ROCKET AXIS PARALLEL TO THE HORIZONTAL INITIAL VELOCITY

The gravitational forces will produce the following downward displacement of the rocket each second:

\[ r = V \sin \beta = \frac{M'}{S'} \cdot \frac{g}{K_C} \cdot V \]  \hspace{1cm} (167)

102 If \( M' = 1; \ g = 10; \ d = 0.00037 \) (at 10 km altitude), \( S' = 20; \ K_C = 1; \ V = 2,260; \ h = 10,000 \), we have \( r = 0.6 \), i.e., the rocket will drop 60 cm per second.

Eliminating \( d \) and \( V \) from (167) (using (111), (161), and (142)), we find

\[ r = \frac{M' \cdot g \cdot h}{d \cdot S' \cdot K_C \cdot \sqrt{2g} \cdot \sqrt{D_1} \cdot h} \]  \hspace{1cm} (168)

The rate of ascent for motion along the tangent can be computed as follows. We have

\[ L = \frac{p}{T} \cdot t^3 \]  \hspace{1cm} (169)

and from (142)

\[ h = \frac{L^2}{D_1} \cdot \]  \hspace{1cm} (170)

Therefore

\[ h = \frac{p^2 \cdot t^4}{4D_1} \cdot \]  \hspace{1cm} (170)

Differentiation gives

\[ \frac{dh}{dt} = \frac{p^2 \cdot t^3}{D_1} \cdot \sqrt{\frac{64}{D_1}} \cdot \sqrt{p} \cdot h^{3/4} \]  \hspace{1cm} (170)

The above relations can be used to compute Table 24, which characterizes the flight of a rocket with initial horizontal velocity.

We see from this table that horizontal flight should be launched from a tower 100 m high or from a cliff with a 45° slope, if \( p = 10 \). If \( p > 10 \), a lower tower may be used. The rocket will first lose altitude, and then move parallel to the ground, finally approaching a tangent to the earth's surface.
TABLE 24.

<table>
<thead>
<tr>
<th>Time of flight, t, sec</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>400</th>
<th>1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, m/sec ( (p = 10) )</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>1,000</td>
<td>2,000</td>
<td>4,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Path length, ( L, \text{km} )</td>
<td>0.5</td>
<td>2</td>
<td>12.5</td>
<td>50</td>
<td>200</td>
<td>800</td>
<td>5,000</td>
</tr>
<tr>
<td>Altitude ( h = \frac{L^2}{D_3} ) ( \text{(approximate)} ), m</td>
<td>0.02</td>
<td>0.32</td>
<td>12.3</td>
<td>197</td>
<td>3,150</td>
<td>50,400</td>
<td>1,970,000</td>
</tr>
<tr>
<td>Rate of ascent</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.003</td>
<td>0.000878</td>
<td>Close to zero</td>
<td></td>
</tr>
<tr>
<td>( \frac{dh}{dt}, \text{m/sec} )</td>
<td>0.008</td>
<td>0.064</td>
<td>0.554</td>
<td>4.43</td>
<td>35.5</td>
<td>283</td>
<td>4,430</td>
</tr>
<tr>
<td>Air density, ( d )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0013</td>
<td>0.000878</td>
<td>Close to zero</td>
<td></td>
</tr>
<tr>
<td>Rate of fall due to gravity and air drag, m/sec</td>
<td>3.85</td>
<td>1.92</td>
<td>0.77</td>
<td>0.385</td>
<td>0.280</td>
<td>53</td>
<td>4.10^{11}</td>
</tr>
<tr>
<td>( d_l )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.48</td>
<td>550</td>
<td>10^{11}</td>
</tr>
</tbody>
</table>

103 14. ASCENDING FLIGHT

Ascending flight, for small inclination angles of the trajectory to the horizon, will require less propellant energy than horizontal flight.

The ascent height above the ground \( h \) is a function of two variables: the angle of inclination

\[
h' = \sin \delta \cdot L
\]

and the sphericity of the earth

\[
h'' = L^2 / D_3
\]

so that

\[
h = h' + h'' = L \sin \delta + \frac{L^2}{D_3} = L \left( \sin \delta + \frac{L}{D_3} \right)
\]

The amount of descent is determined by relations (167) and (168), where the angle represents the trajectory inclination determined by air drag and flight velocity, in general this angle is very small.

The minimum value of \( p \) in ascending motion is given by

\[
p = g \sin \delta.
\]

When a rocket is launched from a mountain, \( p \) should be much higher than \( g \). The table below lists the minimum values of \( p \) satisfying this condition for various angles \( \delta \).

<table>
<thead>
<tr>
<th>Angle ( \delta, \text{deg.} )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, \text{m/sec}^2 ) ...</td>
<td>0.175</td>
<td>0.349</td>
<td>0.523</td>
<td>0.698</td>
<td>0.872</td>
<td>1.05</td>
<td>1.22</td>
<td>1.39</td>
<td>1.56</td>
<td>1.74</td>
</tr>
<tr>
<td>( 10 p ) ........</td>
<td>1.75</td>
<td>3.49</td>
<td>6.23</td>
<td>8.98</td>
<td>8.72</td>
<td>10.5</td>
<td>12.2</td>
<td>13.9</td>
<td>15.6</td>
<td>17.4</td>
</tr>
</tbody>
</table>

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We see from the table that even if $p$ is multiplied tenfold and $\delta = 10^\circ$, the acceleration will only exceed $g$ by a factor of 1.7. For smaller angles, however, smaller propulsion accelerations will be sufficient (about 0.1 of the gravitational acceleration), and this has obvious technical advantages.

Table 25 lists the engine power developed per 1 ton of rocket mass for various $p$ and $V$. The energy is expressed in kg-m and the velocities are in km.

**Table 25.**

<table>
<thead>
<tr>
<th>Velocity</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>101</td>
<td>202</td>
<td>303</td>
<td>505</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>101</td>
<td>202</td>
<td>303</td>
<td>505</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>101</td>
<td>202</td>
<td>303</td>
<td>505</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>202</td>
<td>303</td>
<td>505</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>101</td>
<td>202</td>
<td>303</td>
<td>505</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
</tr>
</tbody>
</table>

We see from the table that the energy of a 1 ton rocket under conditions of minimum acceleration and a small $\delta$ varies from 100 to 1,100 kg-m. A rocket of this mass will attain the escape velocity of 8 km by burning 4 tons of fuel. The burning time for $p = 1$ ($= 0.1 g$) is 800 sec, so that about 0.5 kg of fuel will be burnt each second.

Table 26 lists some data about the mean rate of fuel burning for various $p$, for a 1 ton rocket.

**Table 26.**

<table>
<thead>
<tr>
<th>Fuel reserve, tons</th>
<th>1</th>
<th>4</th>
<th>9</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final velocity, m/sec</td>
<td>3,465</td>
<td>8,045</td>
<td>11,515</td>
<td>17,170</td>
</tr>
<tr>
<td>Burning time, sec</td>
<td>3,465</td>
<td>8,045</td>
<td>11,515</td>
<td>17,170</td>
</tr>
<tr>
<td>Time in hrs</td>
<td>0.96</td>
<td>2.23</td>
<td>3.2</td>
<td>4.8</td>
</tr>
<tr>
<td>Quantity of fuel for $p = 1$, kg</td>
<td>0.29</td>
<td>0.5</td>
<td>0.78</td>
<td>1.75</td>
</tr>
<tr>
<td>Same, for $p = 5$</td>
<td>1.45</td>
<td>2.5</td>
<td>3.9</td>
<td>8.75</td>
</tr>
<tr>
<td>Same, for $p = 10$</td>
<td>2.9</td>
<td>5</td>
<td>7.8</td>
<td>17.5</td>
</tr>
</tbody>
</table>

If the fall velocity (167) is taken equal to the ascent velocity (74), we find

$$\sin\delta = \frac{M'}{d} \cdot \frac{E}{S' K_c V^2}.$$  

With this angle, the initial motion is horizontal. For instance, if $M' = 1$; $g = 10$; $S' = 20$; $K_c = 1$; $V = 100$, we have $\delta = 2.2^\circ$. For $V = 200$, we have $\delta = 0.5^\circ$.  

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15. THE WORK OF SOLAR GRAVITY

If the rocket is to escape from the earth and to become a satellite of the sun, moving in the same orbit as the earth, it will have to be imparted a velocity \( V = 11.17 \text{ km/sec} \) (Figure 38). The rocket will then have an orbital velocity \( V = 29.5 \text{ km/sec} \) around the sun, the same as the earth. If the rocket is to escape from the solar system, it will have to acquire an additional velocity \( V = 29.5 \cdot \sqrt{2} - 29.5 = 12.21 \text{ km/sec} \) in the direction of its motion. The work to escape from the earth is thus \( \frac{mV^2}{2} \), and the work to escape from the sun is \( \frac{mV^2}{2} \), where \( m \) is the mass of the rocket. The total work corresponds to the velocity \( V \), which is obtained from the equation

\[
\frac{mV^2}{2} = \frac{mV_1^2}{2} + \frac{mV_s^2}{2};
\]
or

\[
V = \sqrt{V_1^2 + V_s^2}.
\]

Hence,

\[
V = 16.55 \text{ km/sec}
\]

If the initial velocity was parallel to the earth's equatorial velocity \( V = 0.465 \text{ km/sec} \), the final velocity will be \( V = V - V = 16 \text{ km/sec} \). The rocket will escape from the sun into the Milky Way.

If the rocket is launched against the orbital motion of the earth, a substantially higher velocity will be needed. The rocket will first have to lose the orbital velocity of 29.5 km/sec and then acquire the velocity of

\[
29.5 \cdot \sqrt{2} = 41.7 \text{ km/sec},
\]
so that its total velocity relative to the sun should be

\[ 29.5 + 41.7 = 71.2 \text{ km/sec}. \]

The total velocity will be found, as before, from the equality

\[ V' = \sqrt{71.2^2 + 11.17^2} = \sim 75.1 \text{ km/sec}. \]

The total velocity in this case is higher than the previous velocity \( V \), by a factor \( 75.1/16.55 = 4.5 \), and the work is higher by a factor of \( (4.5)^2 = 20 \).

16. CONCLUSIONS

On the basis of the above treatment, Tsiolkovskii proposed the following method of rocket launching.

The rocket is mounted on an automobile (or an airplane, a dirigible, a gun, etc.) which travels down a 10–20° gradient with a velocity of 40–100 m. The road should be laid in high mountains. The rocket then detaches from the carrier and is launched into the air. The air oxygen is used for combustion. The slope of the trajectory diminishes with increasing velocity, and the trajectory eventually becomes parallel to the ground and the rocket escapes from the earth. In the high rarefied layers of the atmosphere, the air oxygen is not enough for combustion and the rocket engines draw on inner oxygen reserves.

17. A GENERAL PLAN FOR THE CONQUEST OF OUTER SPACE

Tsiolkovskii sketched the following plan for the conquest of outer space. A space station is first built at a distance of 1,000–2,000 km from the earth. This station gradually accumulates large quantities of solar energy for 10^6 long-distance flights. Rockets will tap this energy reserve by means of special conductors.

Rockets are then used to travel to the asteroids, on which landing can be accomplished without much difficulty. The asteroids constitute a source of fuel for travel to other planets and stars. To land on earth, the rockets will describe gradually descending spirals and eventually coast through the air like aircraft. It is best to make the rockets splash in the ocean.

18. FEEDING THE CREW IN FLIGHT

Each rocket should carry certain plants which will purify the air and produce nourishing fruit. One person requires 3,000 large calories daily from food. This corresponds to the energy output of 0.5 kg of coal, or 1 kg of flour, or 3 kg of potatoes, or 2 kg of meat. This energy can be obtained in outer space by utilizing as little as 7% of the solar rays hitting
perpendicularly a plane surface of 1 m². It then remains to transform this solar energy into nourishing substances produced by plants.

19. LAUNCHING A ROCKET FROM EARTH

To facilitate the launch, Tsiolkovskii devised a two-stage rocket, the actual space rocket being carried in the nose of an earth rocket. The reaction force of the propellant drives the earth rocket over level ground until its velocity reaches a certain value. Then the propellant in the space rocket is ignited, the space rocket separates from the carrier and escapes into space.* Figure 39, top, shows the entire two-stage assembly with the space rocket (20 m long, 2 m in diameter) and the earth rocket (100 m long, 107 2 m in diameter) linked together. Inside the earth rocket we see the explosion chamber (1), the pumps (2) which drive the fuel (4) through the engine (3) into the combustion chamber. Figure 39, bottom, shows a longitudinal section through an inclined metallic ramp (10°-20° gradient) with an overall length of up to 500 km, built in the mountains. It is along this ramp that the earth rocket, abundantly lubricated, will move, reaching velocities of 3,260 m/sec. At the end of the ramp, the space rocket will separate, continuing further acceleration by means of its own propellant. To facilitate the ejection of the space rocket, the carrier rocket starts decelerating on contact with the unlubricated portion of the ramp. The deceleration is further enhanced by means of air-drag increasing fins and surfaces which are extended from the rocket at this point.

* This design was quoted by A. Scherschevsky (see p.41).

** Pneumatic "lubrication" is proposed by Tsiolkovskii as a means of reducing friction with the ramp surface (see N.Rynin, "Rockets," p.83 [English translation by IPST, TT 70-50114. 1971]).

The frictional losses on the ramp are estimated by Tsiolkovskii to be 2% of the total kinetic energy. Tsiolkovskii claims that these frictional losses can be reduced virtually to zero.** In his opinion, the air drag does not exceed

$$\frac{d}{d_s} S \cdot V_n$$

(176)
where \( d' \) is the air density, \( g \) is the gravitational acceleration, \( S \) is the friction surface area, \( V_m \) is the velocity of the air molecules.

Tsioolkovskii further maintains that for constant external pressure and fixed gas composition, the friction is proportional to the square root of the molecular weight of the gas and inversely proportional to the square root of the gas temperature.

For air we find from (176) frictional forces of 0.011 ton per 1 sq. m. Tsioolkovskii also gives an alternative formula for air drag:

\[
F_i = \frac{b}{2g} \cdot n \cdot d' \cdot V',
\]

(177)

where \( b \) is the thickness of the air layer adhering to a 1 m\(^2\) area at velocities of 1 m/sec, \( n \) is the width of the contact surface, \( V \) is the velocity of the contact surface.

This relation is valid for \( V = l \) (the length of the rocket); then

\[
F_i = \frac{b}{2g} \cdot V \cdot n \cdot d' .
\]

(178)

For \( g = 20 \), \( b = 3 \), \( d' = 0.0013 \), and \( b = 0.01 \) m,

we have

\[
F_i = 195 \cdot 10^{-8} \cdot V = 195 \cdot 10^{-8} \cdot l^3 .
\]

(179)

108 Let the rocket mass be 1 ton. The results for various accelerations \( p \) and various velocities \( V \) are listed in Table 27.

| TABLE 27. |
| Length, weight, and velocity of earth rocket in m, m/sec, ton | 1 | 10 | 100 | 500 | 1,000 | 1,500 | 2,000 | 3,000 | 5,000 |
| Air friction in kg | 0.002 | 0.2 | 20 | 500 | 2,000 | 4,500 | 8,000 | 18,000 | 50,000 |
| Drag in relation to pressure on projectile, \( \% \) for \( C = 10 \) | 0.0002 | 0.002 | 0.02 | 0.1 | 0.2 | 0.3 | 0.4 | 0.6 | 1 |
| Same, \( C = 1 \) | 0.002 | 0.02 | 0.2 | 1 | 2 | 3 | 4 | 6 | 10 |
| Same, \( C = 4 \) | 0.0005 | 0.005 | 0.05 | 0.25 | 0.5 | 0.75 | 1 | 1.5 | 2.5 |

The length \( l \) of the rocket should not exceed 100 m. If the rocket is shorter by a factor of \( \left( \frac{V}{l} \right) \), the time is reduced by the same factor, but the thickness of the air drag layer is reduced by a factor of \( 1 + \ln \left( \frac{V}{l} \right) \). The air drag is reduced by the same factor, so that equation (178) is replaced by a more exact relation, which is valid for rockets of any length:

\[
F_i = \frac{b}{2g} \cdot n \cdot d' \cdot V^3 \left( 1 + \ln \left( \frac{V}{l} \right) \right).
\]

(180)
Taking $l = 100$ m, we compute the results listed in Table 28 for various $V$.

**Table 28.**

<table>
<thead>
<tr>
<th>Velocity $V$, m</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>700</th>
<th>1,000</th>
<th>2,000</th>
<th>3,000</th>
<th>4,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{V}{l}$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>$\ln \left( \frac{V}{l} \right)$</td>
<td>0</td>
<td>0.69</td>
<td>1.10</td>
<td>1.39</td>
<td>1.61</td>
<td>1.95</td>
<td>2.30</td>
<td>3.00</td>
<td>3.40</td>
<td>3.69</td>
</tr>
<tr>
<td>$\ln \left( \frac{V}{l} \right)$</td>
<td>1</td>
<td>1.69</td>
<td>2.10</td>
<td>2.39</td>
<td>2.61</td>
<td>2.95</td>
<td>3.30</td>
<td>4.00</td>
<td>4.40</td>
<td>4.69</td>
</tr>
</tbody>
</table>

The last line shows by what factor the air-drag layer and the air drag diminish for various $\frac{V}{l}$.

Taking in (180)

$$b = 0.01; \ l = 100; \ n = 3,$$

we find

$$F_v = 195 \cdot 10^{-6} \cdot V \left\{ 1 + \ln \left( \frac{V}{l} \right) \right\}. \quad (181)$$

109 Using this relation, we compute Table 29, which lists the absolute and the relative values of air drag for different accelerations $P$.

**Table 29.**

<table>
<thead>
<tr>
<th>Velocity $V$, m</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, kg</td>
<td>19.5</td>
<td>23.1</td>
<td>27.9</td>
<td>32.6</td>
<td>37.4</td>
</tr>
<tr>
<td>Mass 100 ton, $p = 10$</td>
<td>0.02</td>
<td>0.023</td>
<td>0.028</td>
<td>0.033</td>
<td>0.037</td>
</tr>
<tr>
<td>Mass 100 ton, $p = 1$</td>
<td>0.2</td>
<td>0.23</td>
<td>0.28</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>Mass 10 ton, $p = 1$</td>
<td>2</td>
<td>2.3</td>
<td>2.8</td>
<td>3.3</td>
<td>3.7</td>
</tr>
<tr>
<td>Mass 10 ton, $p = 4$</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Correction:

- numerical values from rows 2–6
- multiplied by 1 2 3 4 5

<table>
<thead>
<tr>
<th>Velocity $V$, m</th>
<th>700</th>
<th>1,000</th>
<th>2,000</th>
<th>3,000</th>
<th>4,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, kg</td>
<td>46.3</td>
<td>59.1</td>
<td>97.5</td>
<td>133.0</td>
<td>167.0</td>
</tr>
<tr>
<td>Mass 100 ton, $p = 10$</td>
<td>0.046</td>
<td>0.059</td>
<td>0.098</td>
<td>0.133</td>
<td>0.167</td>
</tr>
<tr>
<td>Mass 100 ton, $p = 1$</td>
<td>0.46</td>
<td>0.59</td>
<td>0.98</td>
<td>1.33</td>
<td>1.67</td>
</tr>
<tr>
<td>Mass 10 ton, $p = 1$</td>
<td>4.6</td>
<td>5.9</td>
<td>9.8</td>
<td>13.3</td>
<td>16.7</td>
</tr>
<tr>
<td>Mass 10 ton, $p = 4$</td>
<td>1.1</td>
<td>1.5</td>
<td>2.5</td>
<td>3.3</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Correction:

- numerical values from rows 2–6
- multiplied by 7 10 20 30 40

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For the lowest acceleration \((p = 1)\) and very small mass (10 tons), the air friction losses do not exceed 17%.

Let us find the maximum velocities of the earth rocket.

We have

\[
R = p - g \sin \beta
\]  

(182)

Here \(R\) is the resultant force, \(p\) is the acceleration produced by the propellant, and \(\beta\) is the angle of inclination to the horizon.

Now,

\[
V = \sqrt{2R} \cdot L = \sqrt{2 (p - g \sin \beta)} \cdot L.
\]

(183)

The pressure of the propellant on the rocket is given by

\[
P = M_i \frac{p}{g}.
\]

(184)

Here \(M_i\) is the mass of the rocket.

We may approximately take \(\beta = 0\), as this approximation hardly affects the value of \(V\). Table 30 lists the velocities of the rocket for various ramp lengths.

<table>
<thead>
<tr>
<th>Ramp length (L), km</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>500</th>
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</thead>
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<tr>
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<td>447</td>
<td>634</td>
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<td>1,420</td>
<td>3,180</td>
<td>4,470</td>
<td>6,340</td>
<td>7,780</td>
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<td>735</td>
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<td>404</td>
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<td>326</td>
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<td>2,000</td>
<td>2,460</td>
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<td>1,091</td>
<td>1,340</td>
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<tr>
<td>10</td>
<td>45</td>
<td>63</td>
<td>103</td>
<td>142</td>
<td>318</td>
<td>447</td>
<td>634</td>
<td>778</td>
<td>1,030</td>
</tr>
</tbody>
</table>

The time of acceleration of the earth rocket can be found by dividing the final velocity by the acceleration \(p\). For \(V = 3,260\) m/sec, we have \(t = \frac{3,260}{10} = 326\) sec = 5 min 26 sec.

Let us find the weight of the fuel needed for a 10 ton earth rocket with a 10 ton space rocket (total rocket mass, 20 ton). Using the table on p. 71, 89 we find the total fuel reserve in tons for various \(V\). The velocity of the gases is taken equal to 4 km/sec.

These velocities are quite sufficient, and the fuel reserve does not exceed 40 tons. There should be no human crew in the earth rocket, since its deceleration may be quite dangerous during the separation of the space rocket and the braking of the earth stage.
20. MOTION OF THE SPACE ROCKET AFTER SEPARATION

We have

\[
dV = - V_i \cdot \frac{dM}{M_i + M},
\]

\[
V = - V_i \ln (M_i + M) + C.
\]

If the initial velocity of the rocket is \( V_0 \), we have \( M = M_2 \), so that

\[
V_0 = - V_i \ln (M_i + M_2) + C.
\]

Subtracting (3) from (185), we find

\[
V - V_0 = V_i \ln \left[ \frac{M_i + M_2}{M_i + M} \right].
\]

If \( M = 0 \), we obtain the maximum velocity \( V_{\text{max}} \)

\[
V_{\text{max}} = V_0 + V_i \ln \left[ 1 + \frac{M_2}{M_i} \right].
\]

If \( V_0 = 3 \text{ km/sec} \) and we require \( V_{\text{max}} = 8 \text{ km/sec} \), we may take \( V_i = 5 \text{ km/sec} \) and the table on p. 84 then gives \( \frac{M_2}{M_i} = 1.8 \).

For the earth rocket we should have taken \( \frac{M_2}{M_i} = 4 \) in order to achieve \( V_{\text{max}} = 8 \text{ km/sec} \) (again table on p. 84).

From (187) we have

\[
\frac{M_i}{M_i} = 1 - \frac{V_{\text{max}} - V_i}{V_i}.
\]

Using this relation, we draw up Table 32.

We see from this table that the introduction of the earth rocket substantially reduces the weight of the space rocket.
In conclusion, I would like to itemize the production costs of this volume:

1) Composition and printing  2,050 rubles
2) Brasses            300 "
3) Art work            60 "
4) Correspondence      40 "

Total  2,450 rubles

The entire edition comprised 700 copies at a cost of \( \frac{2450}{700} = 3 \text{ r 50} \) k each.
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